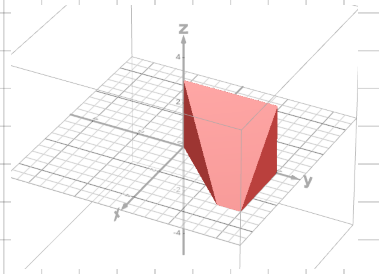
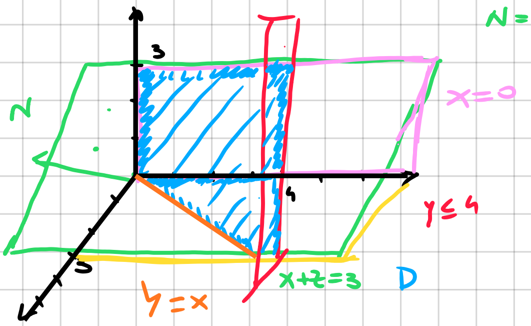
INTEGRALES TRIPLES

* Ejercicio 13 : Graficar D, calcular integrales triples



(a) Volumen $D = \{(x,y,z) \in \mathbb{R}^3 \,/\, \underbrace{x+z \leq 3}_{N=(1,0,1)},\ y \geq x,\ y \leq 4,\ x \geq 0,\ z \geq 0\}$



Hallar límites de integración

$x+z \leq 3 \ \wedge\ z \geq 0 \longrightarrow 0 \leq z \leq 3-x$

$y \leq 4 \ \wedge\ y \geq x \longrightarrow x \leq y \leq 4$

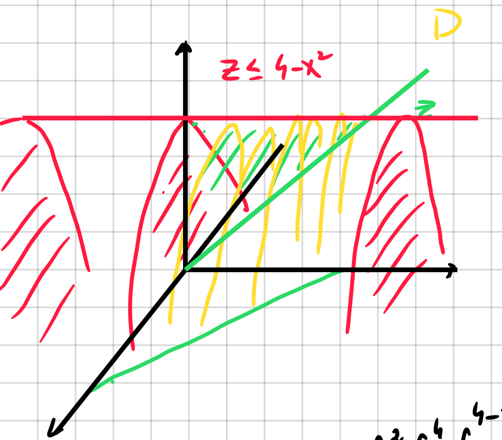
$x \geq 0 \ \wedge\ x \leq 3 \longrightarrow 0 \leq x \leq 3$

$$\text{Volumen } D = \int_0^3 \int_x^4 \int_0^{3-x} dz\,dy\,dx = \int_0^3 \int_x^4 3-x\ dy\,dx = \int_0^3 \left(3y - xy \Big|_x^4 \right) dx = \int_0^3 (12-4x) - (3x + x^2)\,dx$$

$$= \int_0^3 12 - 7x + x^2\,dx = 12x - \frac{7x^2}{2} + \frac{x^3}{3} \Big|_0^3 = \frac{27}{2}$$

(b) Masa D, $D = z \leq 4-x^2,\ z \geq y$, 1° octante $\begin{cases} x \geq 0 \\ y \geq 0 \\ z \geq 0 \end{cases}$, densidad const $\delta(x,y,z) = k$

⇒ Gráfico D



Hallar límite de integración

$z \leq 4-x^2 \ \wedge\ z \geq y \longrightarrow y \leq z \leq 4-x^2$

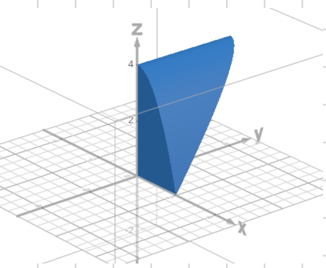
$x \geq 0 \ \wedge\ \underset{\hookrightarrow z=0}{z \leq 4-x^2} \longrightarrow 0 \leq x \leq 2$

$y \geq 0 \ \wedge\ \underset{\hookrightarrow \max z = 4}{z \geq y} \longrightarrow 0 \leq y \leq 4$

$0 \leq y \leq 4-x^2$

$4-x^2$

$$\text{masa}(D) = \int_0^2 \int_0^4 \int_y^{4-x^2} k\,dz\,dy\,dx = k \int_0^2 \int_0^4 \int_y^{4-x^2} dz\,dy\,dx = k \int_0^2 \int_0^4 4-x^2-y\,dy\,dx = k \int_0^2 4y - x^2y - \frac{y^2}{2} \Big|_0^4 dx$$



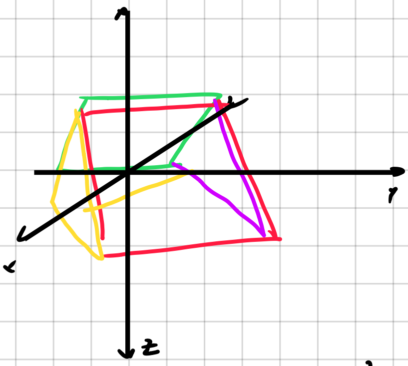
$$\cancel{k \int_0^2 16 - x^2 4 - 8\,dx = k\left(8x - \frac{4x^3}{3} \Big|_0^2 \right) = k\left(16 - \frac{32}{3}\right) = \frac{16k}{3}}$$

$$k \int_0^2 4(4-x^2) - x^2(4-x^2) - \frac{(4-x^2)^2}{2}\,dx = k \int_0^2 (4-x^2)^2 - \frac{(4-x^2)^2}{2}\,dx = k \int_0^2 \frac{1}{2}(4-x^2)^2$$

$$= \frac{k}{2} \int_0^2 16 - 8x^2 + x^4\,dx = \frac{k}{2}\left(16x - \frac{8x^3}{3} + \frac{x^5}{5} \Big|_0^2 \right) = \frac{128}{15} k$$

c) Volumen $D = \{ (x,y,z) \in \mathbb{R}^3 / \ x+y+z \leq 4 \ \wedge \ z \geq x+y \ \wedge \ x \geq 0 \ \wedge \ y \geq 0 \}$

• gráfico D



Hallar límites de integración

$z \geq x+y \ \wedge \ x+y+z \leq 4 \longrightarrow \underbrace{x+y}_{x \geq 0 \wedge y \geq 0 \to 0} \leq z \leq 4-x-y$ dudoso

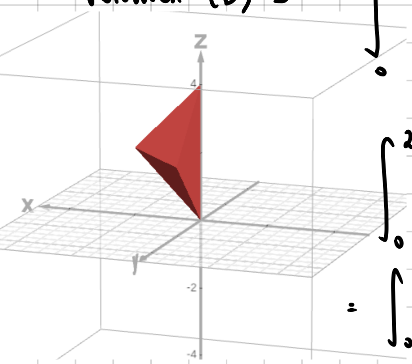
$z \geq x+y \ \wedge \ x+y+z \leq 4 \longrightarrow \underbrace{x+y \leq z \leq 4-x-y}$

$x+y \leq 4-x-y$

$0 \leq y \leq 2-x$

$z \geq x+y \ \wedge \ x+y+z \leq 4 \longrightarrow 0 \leq x \leq 2$ max $y = 2$ ya que $x \geq 0$

$$\text{Volumen } (D) = \int_0^2 \int_0^{2-x} \int_{x+y}^{4-x-y} dz\, dy\, dx = \int_0^2 \int_0^{2-x} 4-x-y-x-y \ dy\, dx = \int_0^2 \left[4y - 2xy - y^2 \Big|_0^{2-x} \right] dx$$



$$\int_0^2 4(2-x) - 2x(2-x) - (2-x)^2 \, dx = \int_0^2 (2-x)\left[4 - 2x - (2-x) \right] dx =$$

$$= \int_0^2 (2-x)(2-x)\, dx = \int_0^2 4 - 4x + x^2 \, dx = 4x - 2x^2 + \frac{x^3}{3} \Big|_0^2 = \frac{8}{3}$$

d) $\iiint_D 3x \, dx\, dy\, dz$. $D: \ z \geq 2x^2 + y^2 + 1, \quad z \leq 5 - y^2$

Hallar lím integración

$z \geq 2x^2 + y^2 + 1, \quad z \leq 5 - y^2$

$2x^2 + y^2 + 1 \leq z \leq 5 - y^2$

$2x^2 + y^2 + 1 \leq 5 - y^2 \longrightarrow \underbrace{2x^2 + 2y^2 \leq 4} \longrightarrow \underbrace{x^2 + y^2 \leq 2}$

Cambio a coord polares

$|y| \leq \sqrt{2-x^2}$, $2 - x^2 \geq 0$

$-\sqrt{2} \leq x \leq \sqrt{2}$

$$\iiint_D 3x\, dx\, dy\, dz = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{2x^2+y^2+1}^{5-y^2} 3x \, dz\, dy\, dx$$

$$\Rightarrow \int_{2x^2+y^2+1}^{5-y^2} 3x\, dz = 3xz \Big|_{2x^2+y^2+1}^{5-y^2} = 3x\left[(5-y^2) - (2x^2+y^2+1) \right] = 3x\left(4 - 2x^2 - 2y^2\right)$$

$$\Rightarrow \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} 3x\left(4 - 2x^2 - 2y^2\right) dy = 3x\left[4y - 2x^2 y - \frac{2y^3}{3} \Big|_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \right] = 3xy\left[4 - 2x^2 - \frac{2y}{3} \Big|_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \right] = 3x\sqrt{2-x^2}\left(4 - 2x^2 - \frac{2\sqrt{2-x^2}}{3}\right)$$

$$- \left(-3x\sqrt{2-x^2}\left(4 - 2x^2 + \frac{2\sqrt{2-x^2}}{3}\right) \right)$$

$$= 3x\sqrt{2-x^2}\left[4 - 2x^2 - \frac{2\sqrt{2-x^2}}{3} + 4 - 2x^2 + \frac{2\sqrt{2-x^2}}{3} \right]$$

$$= 3x\sqrt{2-x^2}\left(8 - 4x^2\right) = 24x\sqrt{2-x^2} - 12x^3\sqrt{2-4x^2}$$

(rehacer punto c)

$$\iiint_D 3x\, dx\,dy\,dz \quad . \quad D: \; z \geq 2x^2+y^2+1, \quad z \leq 5-y^2$$

Hallar lím integración

$z \geq 2x^2+y^2+1, \quad z \leq 5-y^2$

$2x^2+y^2+1 \leq z \leq 5-y^2$

$2x^2+y^2+1 \leq 5-y^2 \longrightarrow \underbrace{2x^2+2y^2 \leq 4}_{\text{Cambio a coord polares}} \longrightarrow \underbrace{x^2+y^2 \leq 2}_{|y| \leq \sqrt{\underbrace{2-x^2}_{2-x^2 \geq 0}}}$

$-\sqrt{2} \leq x \leq \sqrt{2}$

$$\iiint_D 3x\, dx\, dy\, dz = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{2x^2+y^2+1}^{5-y^2} 3x\, dz\, dy\, dx$$

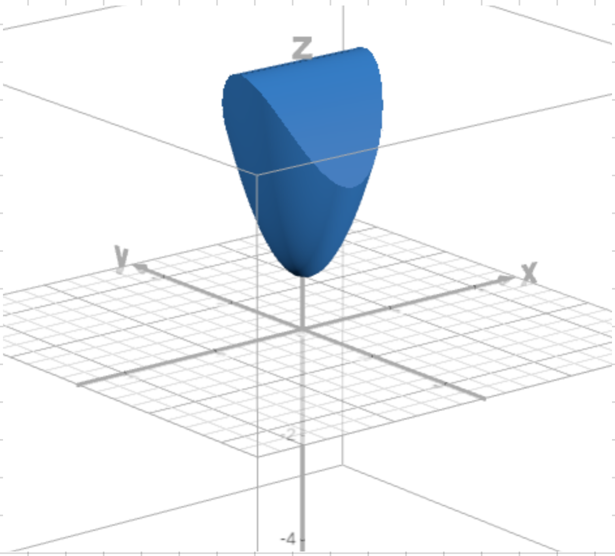
$$= \iint_{x^2+y^2\leq 2} \left(\int_{2x^2+y^2+1}^{5-y^2} 3x\, dz \right) dx\,dy = \iint_{x^2+y^2\leq 2} 3x\left(5-y^2-2x^2-y^2-1\right) dx\,dy$$

$$= \iint_{x^2+y^2\leq 2} 3x\left(4-2x^2-2y^2\right) dx\,dy \qquad \begin{array}{l} x = r\cos(\theta) \\ y = r\,\mathrm{sen}(\theta) \end{array}, \quad \begin{array}{l} 0 \leq r \leq \sqrt{2} \\ 0 \leq \theta < 2\pi \end{array}$$

$$= \int_0^{\sqrt{2}} \int_0^{2\pi} 3r\cos(\theta)\left(4-2(r^2\cos^2(\theta))-2(r^2\mathrm{sen}^2(\theta))\right).r\; d\theta\, dr$$

$$= \int_0^{\sqrt{2}} \int_0^{2\pi} 3r^2\cos(\theta)\left[4-2r^2\left(\cos^2(\theta)+\mathrm{sen}^2(\theta)\right)\right] d\theta\, dr = \int_0^{\sqrt{2}} \int_0^{2\pi} 12r^2\cos(\theta) - 6r^4\cos(\theta)\; d\theta\, dr$$

$$= \int_0^{\sqrt{2}} 12r^2\mathrm{sen}(\theta) - 6r^4\mathrm{sen}(\theta) \Big|_0^{2\pi} dr = \int_0^{\sqrt{2}} 0\; dr = 0$$



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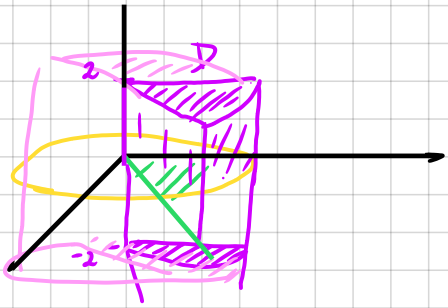
* **Ejercicio 14** : graficar cuerpo en espacio + calcular con coord cilíndrica

(a) Masa $D = \{(x,y,z) \in \mathbb{R}^3 \ / \ x^2+y^2 \leq 4, \ y \geq x, \ |z| \leq 2\}$, $\delta(x,y,z) = Kx^2$

Gráfico D



Coordenadas cilíndricas

$$\begin{cases} x = r\cos(\theta) \\ y = r\,\mathrm{sen}(\theta) \\ z = z \end{cases} \qquad (x,y,z) = h(r,\theta,z)$$

$$J(r,\theta,z) = \left| \begin{matrix} \cos\theta & -r\,\mathrm{sen}\theta & 0 \\ \mathrm{sen}(\theta) & r\cos(\theta) & 0 \\ 0 & 0 & 1 \end{matrix} \right| = r$$

- Hallar lim Integración $\qquad 0 \leq r \leq 2$

$x^2+y^2 \leq 4 \rightarrow r^2(\cos^2\theta + \mathrm{sen}^2\theta) \leq 4$

$y \geq x \rightarrow r\,\mathrm{sen}(\theta) \geq r\cos(\theta), \rightarrow \mathrm{tg}(\theta) \geq 1 \rightarrow \frac{\pi}{4} \leq \theta \leq \frac{5\pi}{4}$

$$\text{Masa } D = \int_{-2}^{2}\int_{0}^{2}\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} K\,(r\cos\theta)^2\, r\, d\theta\, dr\, dz = K \int_{-2}^{2}\int_{0}^{2}\int_{\frac{\pi}{2}}^{\frac{5\pi}{4}} r^2\cos^2\theta \cdot r\, d\theta\, dr\, dz$$

$\cos^2 - \mathrm{sen}^2 = \cos(2\theta)$

$\cos^2 + \mathrm{sen}^2 = 1$

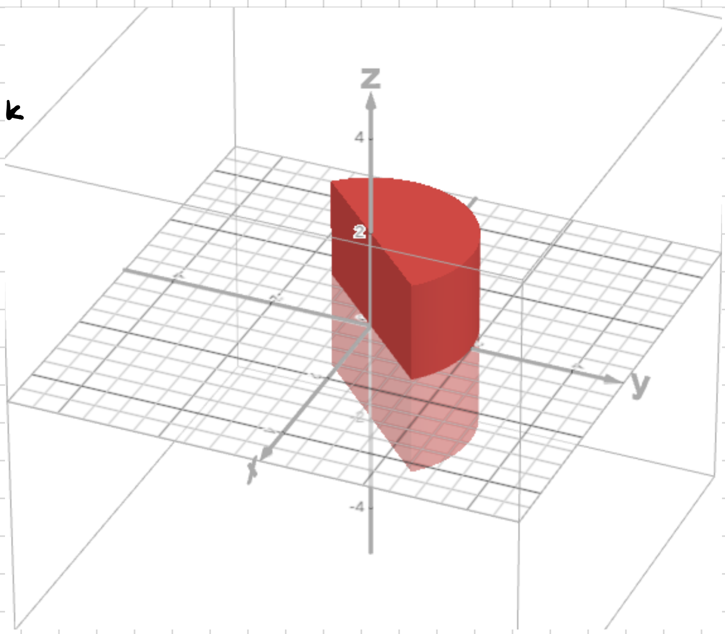
$\cos^2 = \frac{\cos(2\theta)+1}{2}$

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} r^3 \cos^2\theta\, d\theta = r^3 \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \frac{\cos(2\theta)}{2} + \frac{1}{2}\, d\theta = r^3\left(\frac{\mathrm{sen}(2\theta)}{4} + \frac{1}{2}\theta \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\right) =$$

$$= r^3\left[\left(\frac{\mathrm{sen}(\frac{5\pi}{2})}{4} + \frac{5\pi}{8}\right) - \left(\frac{\mathrm{sen}(\frac{\pi}{2})}{4} + \frac{\pi}{8}\right)\right] = r^3\left(\frac{1}{4} + \frac{5\pi}{8} - \frac{1}{4} - \frac{\pi}{8}\right) = \frac{\pi r^3}{2}$$

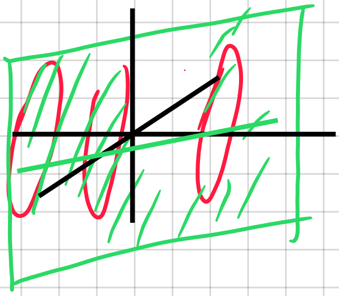
$$\int_0^2 \frac{\pi}{2} r^3\, dr = \frac{\pi}{2}\frac{r^4}{4}\Big|_0^2 = 2\pi$$

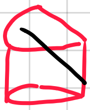
$$\int_{-2}^{2} 2\pi\, dz = 2\pi z \Big|_{-2}^{2} = 4\pi - (-4\pi) = 8\pi K$$



(b) Volumen $D = \{(x,y,z) \in \mathbb{R}^3 / \ x^2+z^2 \leq 9, \ x+y \leq 3, \ y \geq 0\}$

Gráfico D





coordenadas cilíndricas

$$\begin{cases} x = r\cos(\theta) \\ y = y \\ z = r\,\mathrm{sen}(\theta) \end{cases}$$

· Hallar lim integra

$x^2+z^2 \leq 9 \rightarrow r^2 \leq 9 \rightarrow 0 \leq r \leq 3$

$x+y \leq 3, \ y \geq 0 \rightarrow \underbrace{0 \leq y \leq 3-x}$

$0 \leq y \leq 3 - r\cos(\theta)$

$0 \leq 3 - r\cos(\theta)$

$\boxed{\theta \leq \arccos\left(\frac{3}{r}\right)} \rightarrow$ si $r_{max} = 3$, $r_{min} = 0$

$$\int_0^3 \int_0^{2\pi} \int_0^{3-r\cos(\theta)} r \, dy \, d\theta \, dr = \int_0^3 \int_0^{2\pi} 3r - r^2\cos(\theta) \, d\theta \, dr = \int_0^3 \left(3r\theta - r^2\mathrm{sen}(\theta) \Big|_0^{2\pi} \right) dr$$

$$= \int_0^3 6\pi r \, dr = \frac{6\pi r^2}{2} \Big|_0^3 = 27\pi$$

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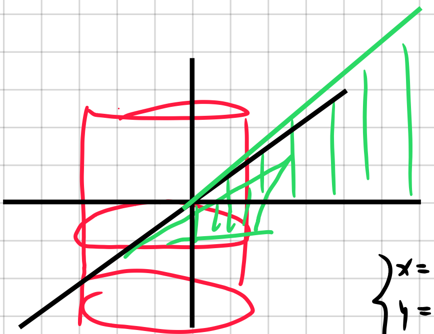
(c) $\iiint_H 2y \, dx\,dy\,dz$, con $H = x^2+y^2 \leq 2x$, $0 \leq z \leq y$



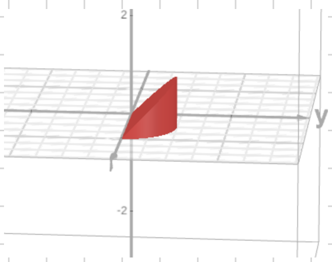
· Ver qué es $x^2+y^2 \leq 2x$

$x^2 - 2x + y^2 \leq 0$

$(x^2 - 2.1x + 1 - 1) + y^2 \leq 0$

$\underbrace{(x-1)^2 + y^2 \leq 1}_{\text{cilindro}}$ centro $= (1, 0, z)$, $r = 1$

$$\begin{cases} x = r\cos(\theta) + 1 \\ y = r\,\mathrm{sen}(\theta) \\ z = z \end{cases}$$



• Hallar lim integración

$0 \leq z \leq y \rightarrow 0 \leq r\,\mathrm{sen}(\theta) \rightarrow 0 \leq \theta \leq \pi$

$x^2+y^2 \leq 2x \rightarrow (r\cos(\theta)+1)^2 + r^2\mathrm{sen}^2 \leq 2r\cos(\theta) + 2$

$r^2(\cos^2 + \mathrm{sen}^2) + 2r\cos(\theta) + 1 \leq 2r\cos(\theta) + 2$

$\boxed{r^2 \leq 1} \rightarrow 0 \leq r \leq 1$

$0 \leq z \leq y \rightarrow 0 \leq z \leq r\,\mathrm{sen}(\theta)$

$$\iiint_H 2y \, dx\,dy\,dz = \int_0^1 \int_0^{\pi} \int_0^{r\,\mathrm{sen}(\theta)} 2r\,\mathrm{sen}(\theta) . r \, dz \, d\theta \, dr$$

$\cos^2 + \mathrm{sen}^2 = 1$

$\cos^2 - \mathrm{sen}^2 = \cos(2x)$

$2\cos^2 = \cos(2x) + 1$

$\cos^2 = \frac{\cos(2x)+1}{2}$

$$\int_0^{r\,\mathrm{sen}(\theta)} 2r\,\mathrm{sen}(\theta)\, r \, dz = 2r^2\mathrm{sen}(\theta) . z \Big|_0^{r\,\mathrm{sen}(\theta)} = 2r^3\mathrm{sen}^2(\theta)$$

$$\int_0^{\pi} 2r^3\mathrm{sen}^2(\theta) \, d\theta = 2r^3 \int_0^{\pi} 1 - \cos^2(\theta) \, d\theta = 2r^3 \int_0^{\pi} \frac{1}{2} - \cos(2\theta) \, d\theta = 2r^3 \left(\frac{1}{2}\theta - \frac{\mathrm{sen}(2\theta)}{2} \right) \Big|_0^{\pi} = \frac{2r^3}{2}\pi = r^3\pi$$

$$\int_0^1 r^3\pi \, dr = \frac{r^4}{4}\pi \Big|_0^1 = \frac{1}{4}\pi$$

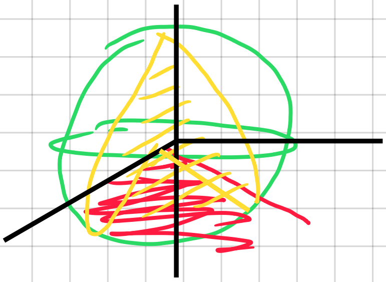
* Ejercicio 15 : resolver en coordenadas esféricas

Coord esférica

$$\begin{cases} x = \rho\cos(\theta)\,\text{sen}(\varphi) \\ y = \rho\,\text{sen}(\theta)\,\text{sen}(\varphi) \\ z = \rho\cos(\varphi) \end{cases}$$

(a) Volumen $D = \{(x,y,z) \in \mathbb{R}^3 \,/\, x^2+y^2+z^2 \le 4,\quad 0 \le y \le x\}$

Gráfico D $\rightarrow$ pedazo de esfera



Hallar límites de integración

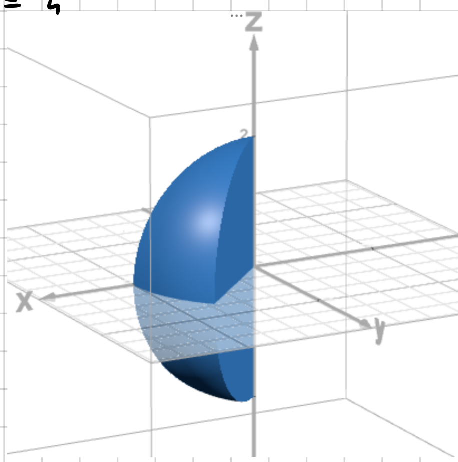
$$x^2+y^2+z^2 \le 4 \rightarrow \rho^2\cos^2(\theta)\,\text{sen}^2(\varphi) + \rho^2\,\text{sen}^2(\theta)\,\text{sen}^2(\varphi) + \rho^2\cos^2(\varphi) \le 4$$

$$\rho^2\left(\cos^2(\theta)\,\text{sen}^2(\varphi) + \text{sen}^2(\theta)\,\text{sen}^2(\varphi) + \cos^2(\varphi)\right) \le 4$$

$$\rho^2\left(\text{sen}^2(\varphi)\left(\cos^2(\theta)+\text{sen}^2(\theta)\right) + \cos^2(\varphi)\right) \le 4 \rightarrow 0 \le \rho \le 2$$

$$0 \le y \le x \rightarrow \rho\,\text{sen}(\theta)\,\text{sen}(\varphi) \le \rho\cos(\theta)\,\text{sen}(\varphi)$$

$$0 \le \text{tg}(\theta) \le 1 \rightarrow 0 \le \theta \le \frac{\pi}{4}$$



$$J(\rho,\theta,\varphi) = \rho^2\,\text{sen}(\varphi)$$

$$\text{Volumen } D = \int_0^2 \int_0^{\frac{\pi}{4}} \int_0^{\pi} \rho^2\,\text{sen}(\varphi)\, d\varphi\, d\theta\, d\rho$$

$$\int_0^{\pi} \rho^2\,\text{sen}(\varphi)\, d\varphi = -\rho^2\cos(\varphi)\Big|_0^{\pi} = \left(-\rho^2(-1)\right) - \left(-\rho^2 \cdot 1\right) = \rho^2+\rho^2 = 2\rho^2$$

$$\int_0^{\frac{\pi}{4}} 2\rho^2\, d\theta = 2\rho^2\theta\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{2}\rho^2 \qquad \int_0^2 \frac{\pi}{2}\rho^2\, d\rho = \frac{\pi}{6}\rho^3\Big|_0^2 = \frac{4\pi}{3}$$

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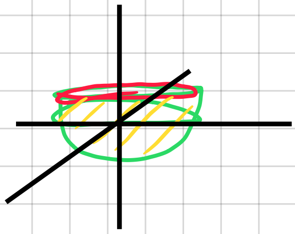
(b) $\iiint_D 2y \; dx\,dy\,dz \qquad D = x^2+y^2+z^2 \le 4,\quad z \le 1$



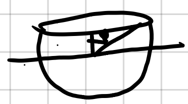
Coord esférica

$$\begin{cases} x = \rho\cos(\theta)\,\text{sen}(\varphi) \\ y = \rho\,\text{sen}(\theta)\,\text{sen}(\varphi) \\ z = \rho\cos(\varphi) \end{cases}$$

$\downarrow$

$0 \le \theta < 2\pi$
plano xy completo

Hallar límite integración

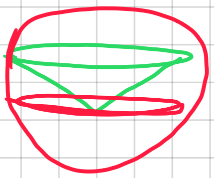


$$x^2+y^2+z^2 \le 4 \rightarrow 0 \le \rho \le 2,$$

$$z \le 1 \rightarrow \rho\cos(\varphi) \le 1$$

$$\cos(\varphi) \le \frac{1}{\rho} \rightarrow \boxed{\varphi \le \arccos\left(\frac{1}{\rho}\right)}$$

$$\varphi \le \frac{\pi}{3}$$



¿cambiar orden integración?

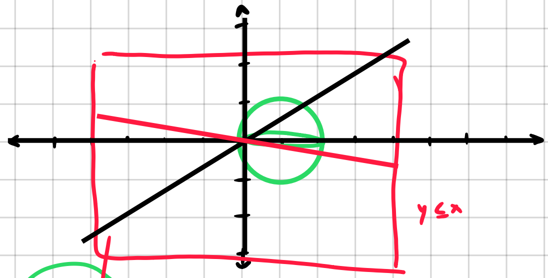
$$\int_0^2 \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\pi} 2\rho\,\text{sen}(\theta)\,\text{sen}(\varphi) \cdot \rho^2\,\text{sen}(\varphi)\, d\varphi\, d\theta\, d\rho =$$

$$\int_{\frac{\pi}{3}}^{\pi} 2\rho^3\,\text{sen}(\theta)\,\text{sen}^2(\varphi)\, d\varphi = 2\rho^3\,\text{sen}(\theta)\int_{\frac{\pi}{3}}^{\pi} \frac{1}{2} - \cos(2\varphi)\, d\varphi = \rho^3\,\text{sen}(\theta)\left(\varphi - \text{sen}(2\varphi)\right)\Big|_{\frac{\pi}{3}}^{\pi} = \rho^3\,\text{sen}(\theta)(\pi) - \rho^3\,\text{sen}(\theta)\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

ⓒ Masa $H = \{(x,y,z) \in \mathbb{R}^3 / \; x^2+y^2+z^2 \leq 2y \; , \; y \leq x\}$ $\qquad \delta = k|z|$

Despejar cuentas

gráfico H

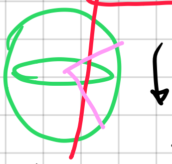


$x^2+y^2+z^2 \leq 2y \qquad x^2+(y-1)^2+z^2 \leq 1$

$x^2+y^2-2y+z^2 \leq 0 \qquad \downarrow$ Esfera centro $(0,1,0)$

coordenadas esféricas

Hallar límite de Integración

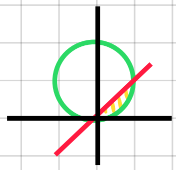


$$\begin{cases} x = \rho\cos(\theta)\,\text{sen}(\varphi) \\ y = \rho\,\text{sen}(\theta)\,\text{sen}(\varphi)+1 \\ z = \rho\cos(\varphi) \end{cases}$$

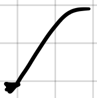
$x^2+y^2+z^2 \leq 2y \rightarrow \quad 0 \leq \rho \leq 1$

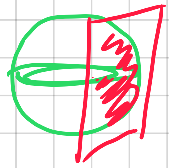
$y \leq x \rightarrow \; \rho\,\text{sen}(\theta)\,\text{sen}(\varphi)+1 \leq \rho\cos(\theta)\,\text{sen}(\varphi)$

$1 \leq \rho\,\text{sen}(\varphi)\left[\cos(\theta)-\text{sen}(\theta)\right]$



y si hago rotación



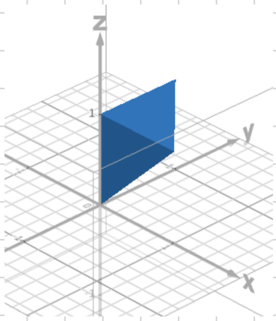


Cálculo integrales triples en coord más conveniente

* Ejercicio 16 : Hallar volumen cuerpo D

ⓐ $D = \{(x,y,z) \in \mathbb{R}^3 / \; \underbrace{x+y \leq z \leq 1}_{\text{planos? tipo caja}} \;\wedge\; \underbrace{x \geq 0 \;\wedge\; y \geq 0}_{\text{borde de cajas}}\}$

Gráfico D



• Hallar lím integración

$x+y \leq z \leq 1 \;\wedge\; x \geq 0 \rightarrow \quad 0 \leq x \leq 1$

$\underbrace{x+y \leq z \leq 1}_{x+y \leq 1} \;\wedge\; y \geq 0 \rightarrow \quad 0 \leq y \leq 1-x$

$$\iiint_D dv = \int_0^1\int_0^{1-x}\int_{x+y}^1 dz\,dy\,dx = \int_0^1\int_0^{1-x} 1-x-y\;dy\,dx = \int_0^1 y-xy-\frac{y^2}{2}\Big|_0^{1-x} dx$$

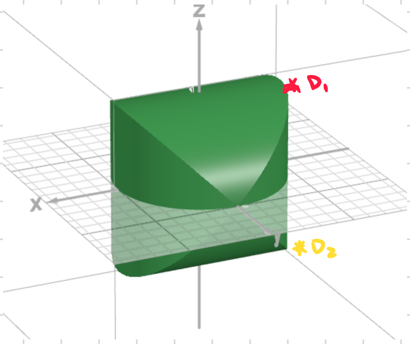
$$= \int_0^1 (1-x)-x(1-x)-\frac{(1-x)^2}{2}\,dx = \int_0^1 (1-x)\left(1-x-\frac{(1-x)}{2}\right)dx = \int_0^1 (1-x)\frac{(1-x)}{2}\,dx$$

$$= \frac{1}{2}\int_0^1 1^2-2x+x^2\,dx = \frac{1}{2}\left(x-x^2+\frac{x^3}{3}\right)\Big|_0^1 = \frac{1}{6}$$

(b) $D = \{(x,y,z) \in \mathbb{R}^3 \,/\, x^2+y^2 \le 1 \ \wedge\ y^2+z^2 \le 1\}$

→ intersección de dos cilindros

gráfico D



• coordenadas cilíndricas

$$\begin{cases} x = r\cos(\theta), \\ y = r\,\text{sen}(\theta) \\ z = z \end{cases}$$

• Hallar lím integración D_1

$x^2+y^2 \le 1 \rightarrow r^2 \le 1 \rightarrow \quad 0 \le r \le 1$

$y^2+z^2 \le 1 \rightarrow r^2\text{sen}^2(\theta) + z^2 \le 1$

$\Downarrow$

$|z| \le \sqrt{1 - r^2\text{sen}^2(\theta)} \rightarrow D$

$$\underbrace{0 \le z \le \sqrt{1-r^2\text{sen}^2(\theta)}}_{\text{lím integr. } D_1}$$

básicamente como la figura es sim

$$V = \iiint_D dv = \iiint_{D_1} dv + \iiint_{D_2} dv = 2\iiint_{D_1} dv \ \Big]$$

si partimos la fig simétrica
el volumen es la suma de cada parte
en este caso $D_1 + D_2 = D$

$$\iiint_{D_1} dv = \int_0^1 \int_0^{2\pi} \int_0^{\sqrt{1-r^2\text{sen}^2(\theta)}} r \, dz\, d\theta\, dr = \int_0^1 \int_0^{2\pi} r\sqrt{1-r^2\text{sen}^2\theta}\, d\theta\, dr = \int$$

$$\int_0^{\sqrt{1-r^2\text{sen}^2(\theta)}} r \, dz = r\sqrt{1-r^2\text{sen}^2(\theta)}$$

rehacer

↓ Se puede calcular por separado para simplificar

osea no aplicar directamente cambio de coord

$$\text{vol} = \iiint_D dv = \iint_{D_{xy}} dx\,dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dz$$

$$\iint_{D_{xy}} \sqrt{1-y^2} - \left(-\sqrt{1-y^2}\right) dx\,dy = 2\iint_{D_{xy}} \sqrt{1-y^2}\, dx\,dy$$

$$\begin{cases} x = r\cos(\theta) \\ y = r\,\text{sen}(\theta) \end{cases}, \quad \begin{matrix} 0 \le r \le 1 \\ 0 \le \theta \le 2\pi \end{matrix}$$

$$2\int_0^1 \int_0^{2\pi} \sqrt{1 - r^2\text{sen}^2(\theta)}\; r\, d\theta\, dr \ \Big]$$

no conviene cambiar a polares
ya que tendría que usar más fórmulas

$$2\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \sqrt{1-y^2}\, dx\, dy = 4\int_{-1}^{1} 1-y^2\, dy = 4\left(y - \frac{y^3}{3}\right)\Big|_{-1}^{1} = 4\left(1-\frac{1}{3}+1-\frac{1}{3}\right) = \frac{16}{3}$$

⇒ Alternativa, usar fórmula volumen steinmetz solid

ⓒ $D = \{(x,y,z) \in \mathbb{R}^3 / \ z \le \sqrt{x^2+y^2} \ \wedge \ x^2+y^2+z^2 \le 2\}$

coordenadas esféricas

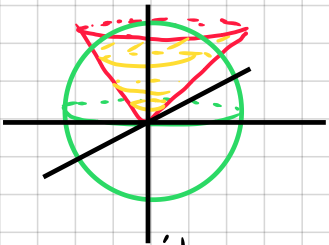
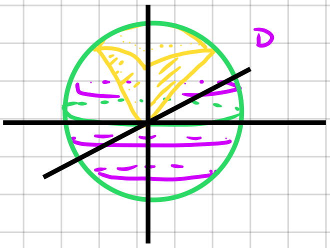


Figura análisis

$$\begin{cases} x = \rho\cos(\theta)\,\mathrm{sen}(\varphi) \\ y = \rho\,\mathrm{sen}(\theta)\,\mathrm{sen}(\varphi) \\ z = \rho\cos(\varphi) \end{cases}, \quad 0 \le \theta < 2\pi$$

por fig análisis



Hallar límites de integración

$x^2+y^2+z^2 \le 2 \longrightarrow \rho \le \sqrt{2} \longrightarrow 0 \le \rho \le \sqrt{2}$

$z \le \underbrace{\sqrt{x^2+y^2}}_{\ge 0} \longrightarrow \rho\cos(\varphi) \le \sqrt{(\rho\cos(\theta)\,\mathrm{sen}(\varphi))^2 + (\rho\,\mathrm{sen}(\theta)\,\mathrm{sen}(\varphi))^2}$

$\rho\cos(\varphi) \le \sqrt{\rho^2\mathrm{sen}^2(\varphi)[\cos^2(\theta)+\mathrm{sen}(\theta)]}$

$\rho\cos(\varphi) \le \rho\,\mathrm{sen}(\varphi)$

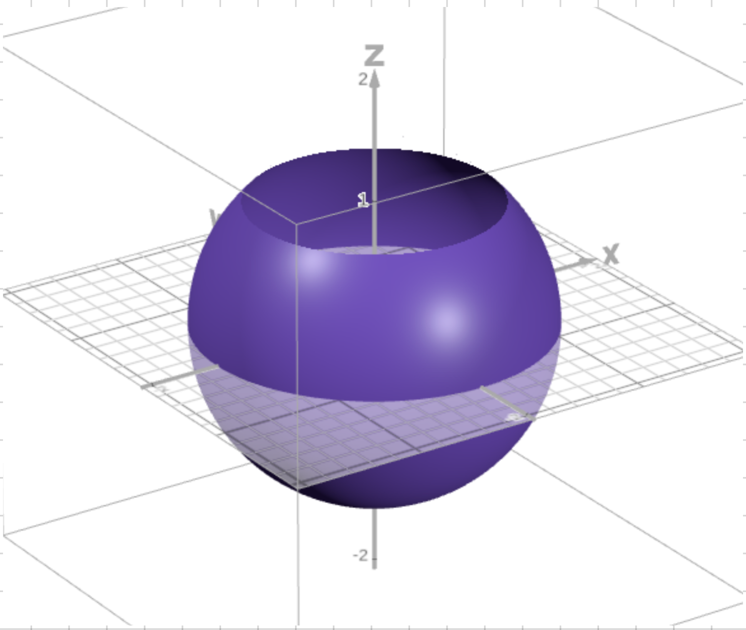
$1 \le \mathrm{tg}(\varphi) \longrightarrow \frac{\pi}{4} \le \varphi \le \pi$

$$\iiint_D dv = \int_0^{\sqrt{2}}\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\pi} \rho^2\,\mathrm{sen}(\varphi)\,d\varphi\,d\theta\,d\rho$$

$$\int_{\frac{\pi}{4}}^{\pi} \rho^2\mathrm{sen}(\varphi)\,d\varphi = -\rho^2\cos(\varphi)\Big|_{\frac{\pi}{4}}^{\pi} = \left(-\rho^2\cos(\pi)\right) - \left(-\rho^2\cos(\tfrac{\pi}{4})\right) = \rho^2 + \frac{\sqrt{2}}{2}\rho^2 = \rho^2\left(1+\frac{\sqrt{2}}{2}\right)$$

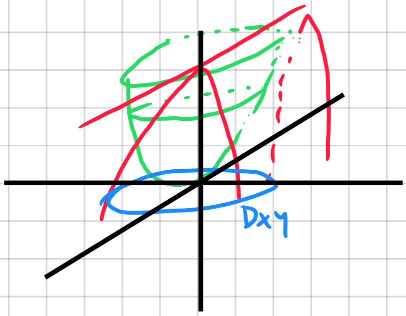
$$\int_0^{2\pi} \rho^2\left(1+\frac{\sqrt{2}}{2}\right)d\theta = \rho^2\left(\theta + \frac{\sqrt{2}}{2}\theta\right)\Big|_0^{2\pi} = \rho^2(2\pi + \sqrt{2}\,\pi)$$

$$\int_0^{\sqrt{2}} \rho^2(2\pi+\sqrt{2}\,\pi)\,d\rho = (2\pi+\sqrt{2}\,\pi)\frac{\rho^3}{3}\Big|_0^{\sqrt{2}} = \frac{2\sqrt{2}}{3}(2\pi+\sqrt{2}\,\pi) = \frac{4\sqrt{2}\,\pi}{3} + \frac{4\pi}{3} = \frac{4\pi}{3}(\sqrt{2}+1)$$



(d) D limitado por $\underbrace{z = 2x^2+y^2}_{\text{paraboloide}}$, $\underbrace{z+y^2=8}_{\text{parábola } x=0}$

Figura de análisis



como $\underline{\text{D limitado}}$ entonces $z \geq 2x^2+y^2 \wedge z \leq 8-y^2$

Como $x^2 \wedge y^2$ es mejor utilizar coordenadas polares

↳ Intersección

$$z = 2x^2+y^2, \quad z = 8-y^2 \longrightarrow 2x^2+y^2 = 8-y^2 \rightarrow x^2+y^2=4$$

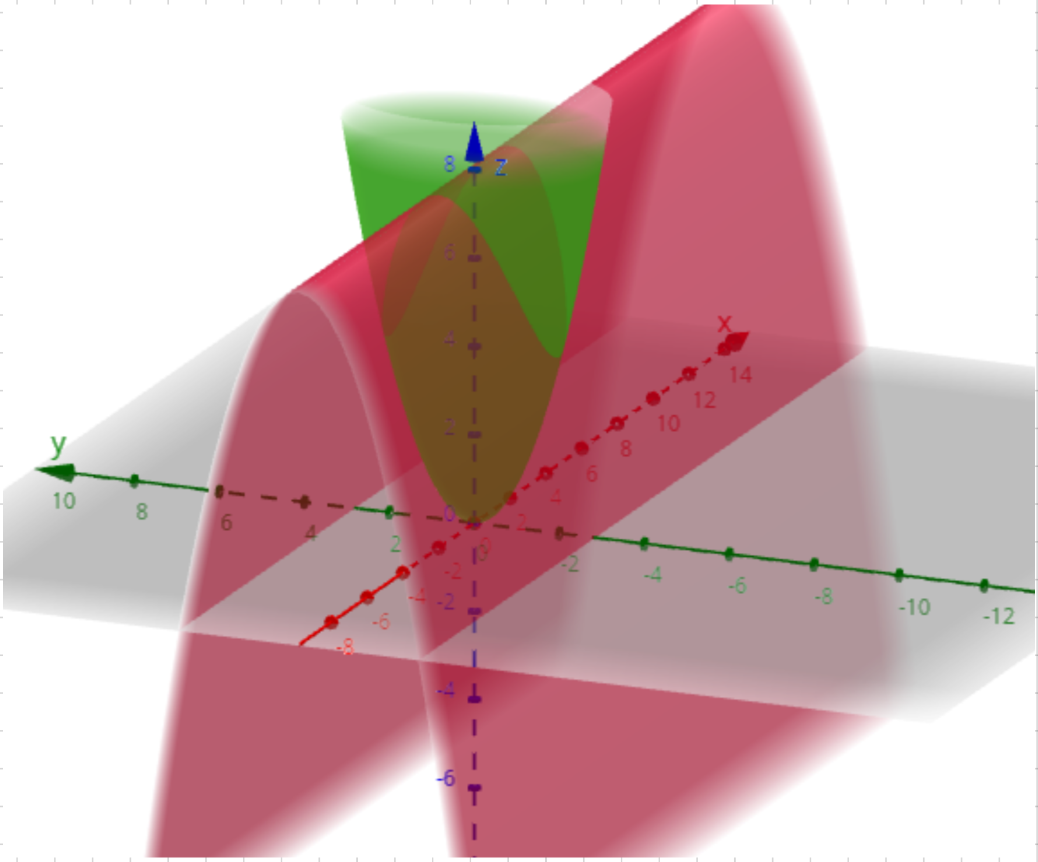
↳ $2x^2+y^2 \leq z \leq 8-y^2 \longrightarrow \underbrace{x^2+y^2 \leq 4}_{Dxy}$ $\quad \begin{cases} x = r\cos(\theta) \\ y = r\,\text{sen}(\theta) \\ z = z \end{cases}$, $\quad 0 \leq r \leq 2$, $\quad 0 \leq \theta \leq 2\pi$

$$\text{Volumen} = \iint_{Dxy} dxdy \int_{2x^2+y^2}^{8-y^2} dz = \iint_{Dxy} 8-y^2-2x^2-y^2 \; dxdy$$

$$\int_0^2 r \int_0^{2\pi} 8 - 2r^2\cos^2(\theta) - 2r^2\text{sen}^2(\theta) \; d\theta \, dr$$

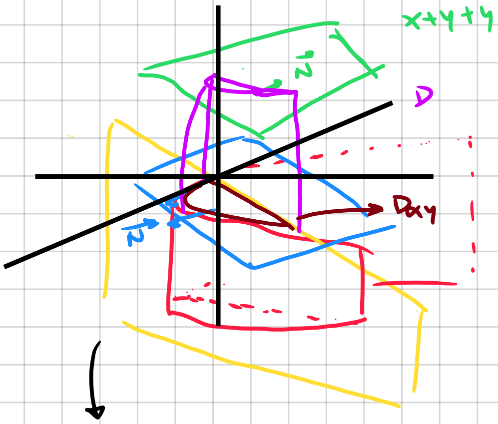
$$\int_0^{2\pi} 8 - 2r^2\left[\cos^2(\theta)+\text{sen}^2(\theta)\right] d\theta = 8\theta - 2r^2\theta \Big|_0^{2\pi} = 16\pi - 4\pi r^2$$

$$\int_0^2 16\pi r - 4\pi r^3 \, dr = 8\pi r^2 - \pi r^4 \Big|_0^2 = 32\pi - 16\pi = 16\pi$$

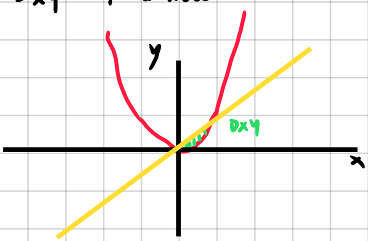


ⓔ D definido por: $y \ge x^2$, $y \le x$, $z \ge x+y$, $x+y+z \le 6$

Figura de análisis (aprox D)



Dxy, donde $z=0$ Fig 2



Hallar límites de integración

$z \ge x+y$, $x+y+z \le 6$ → $x+y \le z \le 6-x-y$

$y \ge x^2$, $y \le x$ → $x^2 \le y \le x$

→ Hallar límite de integración de x por fig análisis 2

$y \ge x^2$, $y \le x$ → $x^2 \le y \le x$ → $x^2 - x \le 0$ $\quad x(x-1) \le 0$

$0 \le x \le 1$

⇒ Hallar Volumen D

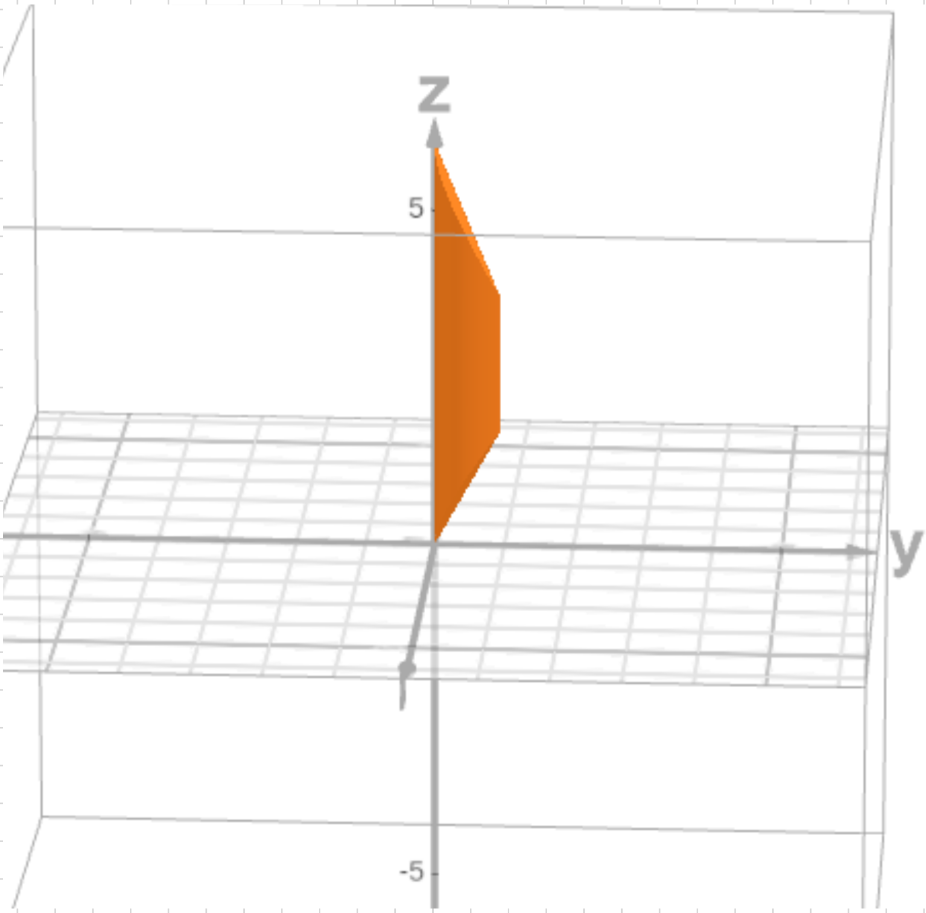
$$\iiint_D dv = \int_0^1 \int_{x^2}^{x} \int_{x+y}^{6-x-y} dz\,dy\,dx$$

$$\int_{x+y}^{6-x-y} dz = 6-x-y-x-y = 6-2x-2y$$

$$\int_{x^2}^{x} 6-2x-2y\;dy = 6y-2xy-\frac{2y^2}{2}\Big|_{x^2}^{x} = \left(6x-2x^2-x^2\right)-\left(6x^2-2x^3-x^4\right)$$

$x^4+2x^3-9x^2+6x$

$$\int_0^1 x^4+2x^3-9x^2+6x\;dx = \frac{x^5}{5}+\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{6x^2}{2}\Big|_0^1 = \frac{x^5}{5}+\frac{x^4}{2}-3x^3+3x^2 = \frac{7}{10}$$



(f) D interior de esfera $x^2+y^2+z^2=\overset{(2r)^2}{4r^2}$. con $x^2+y^2 \geq 2rx$ en 1° octante $\nearrow$ $x \geq 0$, $y \geq 0$, $z \geq 0$

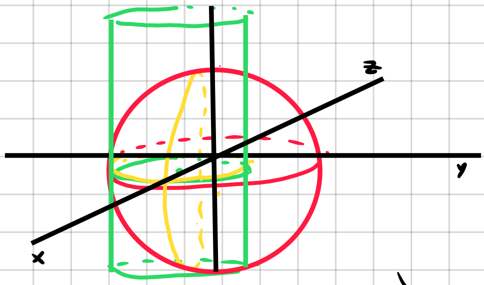
↳ Despejar $x^2+y^2 \leq 2rx$

Figura analisis $r=1$

$$x^2+y^2 \leq 2rx$$

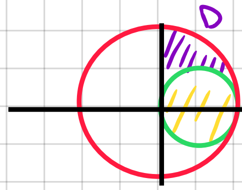
$$x^2-2rx+y^2 \leq 0$$

$$(x-r)^2+y^2 \leq r^2$$



• coordenadas esféricas

$$\begin{cases} x = \rho\cos(\theta)\,\mathrm{sen}(\varphi) \\ y = \rho\,\mathrm{sen}(\theta)\,\mathrm{sen}(\varphi) \\ z = \rho\cos(\varphi) \end{cases}$$



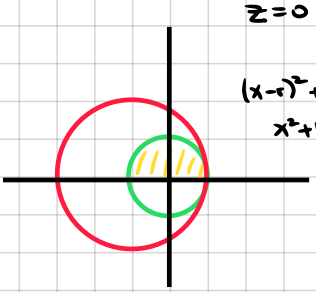
Vista $z=0$

$\rightarrow$ 1er oct $0 \leq \theta \leq \pi$

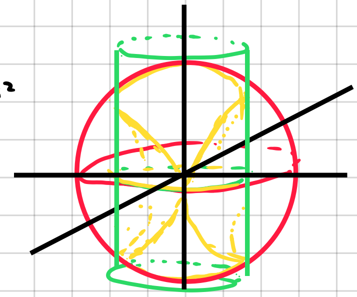
↳ ¿no es mejor correr figura y calcular con coord cilíndricas?

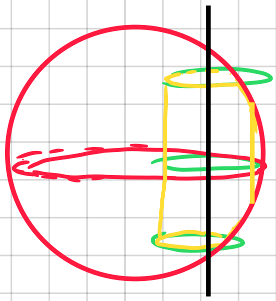
• Mover eje coordenada

$z=0$



$(x-r)^2+y^2+z^2=4r^2$
$x^2+y^2 \leq r^2$





$$\boxed{\mathrm{vol}(D) = \mathrm{vol\ esfera} - \mathrm{vol\ medio\ cilindro}}$$

$$\mathrm{vol}(D) = \frac{\mathrm{vol\ esfera}}{8} - \mathrm{vol\ medio\ cilindro}$$

• coord cilíndrica

$$\begin{cases} x = \rho\cos(\theta) \\ y = \rho\,\mathrm{sen}(\theta) \\ z = z \end{cases},$$

$\underbrace{0 \leq \theta \leq \pi}_{\text{primer oct}}$

• Hallar límites de integración

$x^2+y^2 \leq r^2 \rightarrow \rho^2 \leq r^2 \rightarrow 0 \leq t \leq r$

$(x-r)^2+y^2+z^2 \leq 4r^2 \rightarrow -\sqrt{4r^2-y^2-(x-r)^2} \leq z \leq \sqrt{4r^2-y^2-(x-r)^2}$

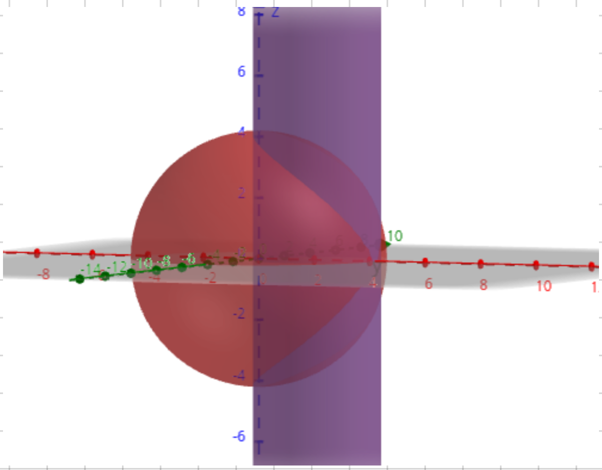
$x^2+y^2 \leq r$

$y \leq \sqrt{r-x^2}$

• Hallar volumen

$$\iiint_D dv = \iint_{D_{xy}} dx\,dy \int_0^{\sqrt{4r^2-y^2-(x-r)^2}} dz = \iint_{D_{xy}} \sqrt{4r^2-y^2-(x-r)^2}\,dx\,dy$$

$$= \int_{-1}^{1}\int_0^{\sqrt{r-x^2}} \sqrt{4r^2-y^2-(x-r)^2}\,dy\,dx$$

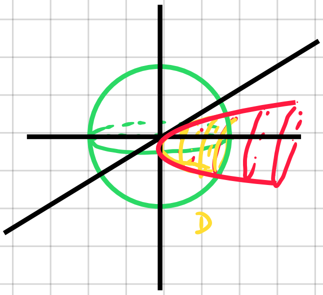


$u = 4r^2-y^2-(x-r)^2$
$du = 2y\,dy$

$\boxed{17/6}$

* **Ejercicio 18** Calcular masa $D: x^2+y^2+z^2 \leq 2$, $y \geq x^2+z^2$, $\delta(x,y,z) = k\sqrt{[(x,y,z)-(0,y,0)]^2} = k\sqrt{x^2+z^2}$

Fig aprox D — en este caso como hay $y \geq x^2+z^2 \to$ conviene utilizar cilíndricas



$$\begin{cases} x = r\cos(\theta) \\ y = y \\ z = r\,\mathrm{sen}(\theta) \end{cases}$$

$D_{xz} = y \geq x^2+z^2$

Hallar lim integración

$x^2+y^2+z^2 \leq 2$. $y^2 \geq x^2+z^2 \to \quad x^2+z^2 \leq y \leq \sqrt{2-x^2-z^2}$

$x^2+z^2 \leq y \leq \sqrt{2-(x^2+z^2)} \longrightarrow x^2+z^2 \leq \sqrt{2-x^2-z^2}$

$r^2\cos^2 + r^2\mathrm{sen}^2 \leq \sqrt{2-r^2}$

$r^2 \leq \sqrt{2-r^2} \to r^4 \leq 2-r^2$

$r^4+r^2-2 \leq 0 \qquad t=r^2$

$t^2+t-2 \leq 0$

$(t+1)(t-2) \leq 0$

$(t+1)(t-2) \leq 0$

$\underbrace{(r^2+1)}_{\geq 0}\underbrace{(r^2-2)}_{\leq 0} \leq 0$

$r^2-2 \leq 0$

$|r| \leq \sqrt{2} \to 0 \leq r \leq \sqrt{2}$

$|J(r,\theta,y)| = r$

• Calcular masa

$$\text{Masa} = \iiint_D \delta\, dv = \iint_{D_{xz}} r\,dxdz \int \delta(x,y,z)\, dy = \iint_{D_{xz}} r\,dxdz \int_{r^2}^{\sqrt{2-r^2}} dy = \int_0^{\sqrt{2}}\int_0^{2\pi} d\theta\, dr \int$$

① $\displaystyle\int_{r^2}^{\sqrt{2-r^2}} dy = \sqrt{2-r^2} - r^2$ ② $\displaystyle\int_0^{\sqrt{2}} r\left(\sqrt{2-r^2}-r^2\right) dr = \frac{1}{2}\int_0 \sqrt{2-\mu} - \mu\, d\mu = \frac{1}{2}\left(-\int t^{\frac{1}{2}} dt - \int \mu\, d\mu\right)$

$\mu = r^2 \qquad t = 2-\mu$

$d\mu = 2r\,dr \qquad dt = -d\mu$

$$= \frac{1}{2}\left(-\frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{\mu^2}{2}\right) = \frac{1}{2}\left(\frac{-2}{3}\sqrt{(2-r^2)}^3 - \frac{r^4}{2}\right)\Bigg|_0^{\sqrt{2}}$$

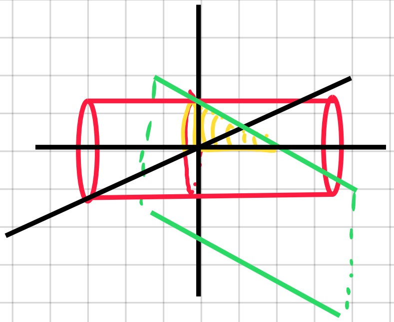
$$= \frac{1}{2}\left(\frac{-2}{3}\cdot 0 - \frac{(\sqrt{2})^4}{2}\right) - \frac{1}{2}\left(-\frac{2}{3}\sqrt{2^3} - 0\right) =$$

$$= (-1) + \frac{2\sqrt{2}}{3}$$ error da negativo

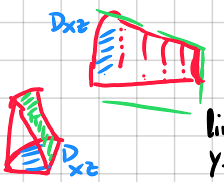
$$\int_0^{2\pi} \frac{2\sqrt{2}}{3} - 1\, d\theta = \frac{2\sqrt{2}}{3}\theta - \theta\Big|_0^{2\pi} =$$

* Ejercicio 20 Calcular coord centro grav $\boxed{\delta = k}$

Cuerpo limitado $x^2+z^2=1$, $y-x=1$, 1° oct $(x\geq 0,\ y\geq 0,\ z\geq 0)$



figura



variable "y" limitado por $y-x=1 \wedge y=0$ plano

$$\iint_{D_{xz}} dx\,dz \int_0^{x+1} \delta \cdot dy = \underbrace{\int_0^1\int_0^{\frac{\pi}{2}} r.d\theta dr \int_0^{x+1} \delta\, dy}_{\text{masa}}$$

• Hallar D_{xz}

$x^2+z^2 \leq 1$ corte con $y=0$ $\left\{\begin{array}{l} x=r\cos(\theta)\\ z=r\operatorname{sen}(\theta)\end{array}\right.$ $0\leq r\leq 1$

$x\geq 0 \quad z\geq 0$

$0\leq x\leq 1 \qquad 0\leq z\leq \sqrt{1-x^2}$

• Hallar masa

$$= \int_0^1\int_0^{\frac{\pi}{2}} r.d\theta dr \int_0^{x+1}\delta\, dy = k\int_0^1\int_0^{\frac{\pi}{2}} r(x+1)\, d\theta\, dr = k\int_0^1\int_0^{\frac{\pi}{2}} r^2\cos(\theta)+r\, d\theta\, dr \cdots k\int_0^1\left(r^2\operatorname{sen}\theta + r\theta\Big|_0^{\frac{\pi}{2}}\right)dr = k\int_0^1 r^2+\frac{\pi}{2}r\, dr$$

$$= k\int_0^1 r^2+\frac{\pi r}{2}\, dr = k\left(\frac{r^3}{3}+\frac{\pi r^2}{4}\right)\Bigg|_0^1 = k\left(\frac{1}{3}+\frac{\pi}{4}\right)$$

• Hallar centro de masa $C_M = (x_G, y_G, z_G)$

$\Rightarrow$ buscar x_G

$$\iint_{D_{xz}} dx\,dz\int_0^{x+1} kx\, dy = k\int_0^1\int_0^{\frac{\pi}{2}} x(x+1).r\, dr\, d\theta = k\int_0^1\int_0^{\frac{\pi}{2}} r^2\cos(\theta)(r\cos\theta+1)\, d\theta\, dr$$

$$= k\int_0^1\int_0^{\frac{\pi}{2}} r^3\cos^2(\theta) + r^2\cos\theta\, d\theta dr = k\int_0^1 \frac{r^3}{2}\theta + r^3\operatorname{sen}(2\theta) + r^2\operatorname{sen}(\theta)\Bigg|_0^{\frac{\pi}{2}} dr$$

$$x_G = \frac{k\left(\frac{\pi}{16}+\frac{1}{3}\right)}{k\left(\frac{1}{3}+\frac{\pi}{4}\right)} \qquad = k\int_0^1 \frac{r^3}{2}\frac{\pi}{2} + \cdot r^2 dr = \frac{\pi r^4}{16}+\frac{r^3}{3}\Bigg|_0^1 = k\left(\frac{\pi}{16}+\frac{1}{3}\right)$$

$\Rightarrow$ Buscar y_G

$$k\iint_{D_{xz}} dx\,dz\int_0^{x+1} y\, dy = k\int_0^1\int_0^{\frac{\pi}{2}} \frac{(r\cos\theta+1)^2}{2}\, d\theta\, dr = \frac{k}{2}\int_0^1\int_0^{\frac{\pi}{2}} r^2\cos^2(\theta) + 2r\cos\theta + 1\, d\theta\, dr = \frac{k}{2}\int_0^1 r\int_0^{\frac{\pi}{2}} r^2\left(\frac{1}{2}+\frac{\cos(2\theta)}{2}\right)+2r\cos\theta+1\, d\theta\, dr$$

$$\frac{k}{2}\int_0^1 r\left(\frac{r^2}{2}\theta + r\operatorname{sen}(2\theta) + 2r\operatorname{sen}(\theta)+\theta\right)\Bigg|_0^{\frac{\pi}{2}} dr = \frac{k}{2}\int_0^1 \frac{r^3\pi}{4} + 2r^2 + \frac{\pi}{2}r\, dr = \frac{k}{2}\left(\frac{r^4\pi}{16}+\frac{2r^3}{3}+\frac{\pi}{4}r^2\right)\Bigg|_0^1$$

$$y_G = \left(\frac{5\pi}{16}+\frac{2}{3}\right)\Big/\left(\frac{1}{3}+\frac{\pi}{4}\right) \qquad k\left(\frac{5\pi}{16}+\frac{2}{3}\right) \longleftarrow \frac{k}{2}\left(\frac{\pi}{16}+\frac{2}{3}+\frac{\pi}{4}\right)$$

– Buscar z_G

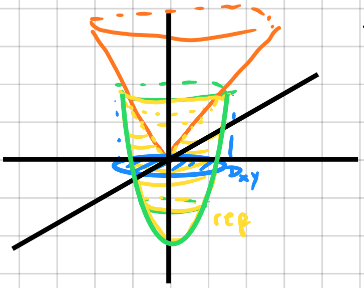
$$\iint_{D_{xz}} dx\,dz\int_0^{x+1} kz\, dy = k\int_0^1\int_0^{\frac{\pi}{2}} r\, z(x+1)\, d\theta dr = k\int_0^1\int_0^{\frac{\pi}{2}} r\, r\operatorname{sen}\theta(r\cos\theta+1)\, d\theta dr = k\int_0^1 r\int_0^{\frac{\pi}{2}} r^2\operatorname{sen}\theta\cos\theta + r\operatorname{sen}(\theta)\, d\theta\, dr$$

$$= k\int_0^1\left(r^3\frac{\operatorname{sen}^2\theta}{2} - r^2\cos(\theta)\right)\Bigg|_0^{\frac{\pi}{2}} dr = k\int_0^1 \frac{r^3}{2} - \left(-r^2\right) dr = k\int_0^1 \frac{r^3}{2}+r^2\, dr = k\left(\frac{r^4}{8}+\frac{r^3}{3}\right)\Bigg|_0^1$$

$$z_G = \left(\frac{11}{24}\right)\Big/\left(\frac{1}{3}+\frac{\pi}{4}\right) \qquad k\left(\frac{1}{8}+\frac{1}{3}\right)$$

* **Ejercicio 21** : Hallar volumen región $x^2+y^2-6 \le z \le \sqrt{x^2+y^2}$

Gráf aprox región



la variable z está def. entre el cono y el paraboloide

Dxy está definida como intersección del cono y del paraboloide (arcant) pero con proyección en $z=0$

$x^2+y^2-6 \le z \le \sqrt{x^2+y^2}$ → caso máximo $z=\sqrt{x^2+y^2}$, $z = x^2+y^2-6$ intersecc → $z \ge 0$

$z=\sqrt{z+6} \to z^2-z-6=0 \to z=-2 \lor \underline{z=3}$ → Intersección

Hallar Dxy

$x^2+y^2-6=3$

$\boxed{x^2+y^2=9} \longrightarrow \begin{cases} x=r\cos(\theta) \\ y=r\operatorname{sen}(\theta)\end{cases}$ $\quad 0\le r\le 3$, $\quad 0\le\theta\le 2\pi$

$$\text{Volumen} = \iint_{Dxy} dx\,dy \int_{x^2+y^2-6}^{\sqrt{x^2+y^2}} dz = \iint_{Dxy} \sqrt{x^2+y^2}-(x^2+y^2)+6 \; dx\,dy$$

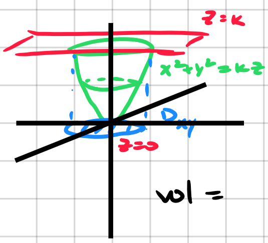
$$\int_0^3\int_0^{2\pi} r(r-r^2+6)\,d\theta\,dr = \int_0^3 r(r\theta-r^2\theta+6\theta)\Big|_0^{2\pi} dr = \int_0^3 r^2 2\pi - r^3 2\pi + 6\cdot 2\pi r\,dr$$

$$= 2\pi\left(\frac{r^3}{3}-\frac{r^4}{4}+3r^2\right)\Big|_0^3 = 2\pi\left(9-\frac{81}{4}+27\right) = \frac{63}{2}\pi$$

$\boxed{18/6}$

* **Ejercicio 22** . Hallar $k>0$ / vol $=4\pi$ entre paraboloide $x^2+y^2=kz$, $z=k$

Figura análisis



$$\text{Volumen} = \iint_{Dxy} dx\,dy \int_0^k dz$$ → 0,0 → z va desde z=k hasta paraboloide. no importa si es (0,0,0) pero es el paraboloide

→ Dxy es la intersección $x^2+y^2=kz$ con $z=k$ → $x^2+y^2=k^2$ $\quad \begin{cases} x=r\cos(\theta) \\ y=r\operatorname{sen}(\theta)\end{cases}$, $0\le\theta\le 2\pi$, $0\le r\le k$

$$\text{vol} = \int_0^k\int_0^{2\pi}\int_0^k dz\,dy\,dx = \int_0^k\int_0^{2\pi} k\,dy\,dx = \int_0^k 2\pi k\,dx = 2\pi k^2$$

$\text{Vol} = 2\pi k^2 = 4\pi$

$k^2 = 2 \to \underbrace{k=\sqrt{2}}_{k>0} \lor k=-\sqrt{2}$

→ corrección

$$\iint_{Dxy} dx\,dy\int_{\frac{x^2+y^2}{k}}^{k} dz = \int_0^k\int_0^{2\pi}\left(k-\frac{x^2+y^2}{k}\right).r\,d\theta\,dr = \int_0^k 2\pi\left(kr-\frac{r^3}{k}\right)dr = 2\pi\left(\frac{kr^2}{2}-\frac{r^4}{4k}\right)\Big|_0^k = 2\pi\left(\frac{k^3}{2}-\frac{k^4}{4k}\right) = 4\pi$$

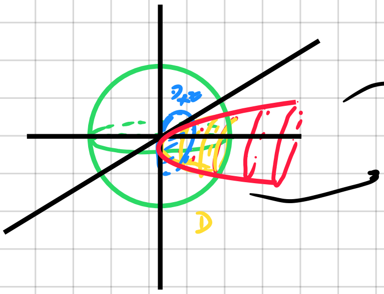
$\frac{k^3}{2}-\frac{k^3}{4} = 2$

$k^3\left(\frac{1}{2}-\frac{1}{4}\right)=2$

$k=\sqrt[3]{8}=2$

⇒ Rehacer ejercicio 18   Calcular masa $D: x^2+y^2+z^2 \le 2$, $y \ge x^2+z^2$, $\delta(x,y,z) = k\sqrt{[(x,y,z)-(0,y,0)]^2} = k\sqrt{x^2+z^2}$

Fig aprox D



$$masa = \iint_{D_{xz}} dx\,dz \int_{x^2+z^2}^{\sqrt{2-(x^2+z^2)}} \delta \, dy$$

→ y varía entre esfera y paraboloide → $x^2+z^2 \le y \le \sqrt{2-(x^2+z^2)}$

D_{xz} es la intersección entre el paraboloide y la esfera

$x^2+y^2+z^2=2$ $y=x^2+z^2$

$y^2+y=2$ $y^2+y-2=0$ $\frac{-1\pm\sqrt{1^2-4.1(-2)}}{2} =$ $y=1$ (✓) $y=-2$ (x)

Intersección $\boxed{x^2+z^2=1}$ → $\begin{cases} x=r\cos(\theta) \\ z=r\,\text{sen}(\theta) \end{cases}$, $0 \le r \le 1$, $0 \le \theta \le 2\pi$

$$masa = \int_0^1 \int_0^{2\pi} \int_{x^2+z^2}^{\sqrt{2-(x^2+z^2)}} k\sqrt{x^2+z^2}\, dy = k \int_0^1 \int_0^{2\pi} \sqrt{x^2+z^2}\left(\sqrt{2-(x^2+z^2)} - (x^2+z^2)\right) r \, d\theta\, dr$$

$$k\int_0^1\int_0^{2\pi} \sqrt{r^2}\left(\sqrt{2-r^2} - r^2\right).r \, d\theta\, dr = k\int_0^1 r^2\left(\sqrt{2-r^2}\,\theta - r^2\theta\right)\Big|_0^{2\pi} dr$$

$$2\pi k \int_0^1 r^2\sqrt{2-r^2} - r^4 \, dr$$

$$2\pi k\left(-\frac{r}{4}(2-r^2)^{\frac{3}{2}} + \frac{1}{4} r\sqrt{2-r^2} + \frac{1}{2}\arcsen\left(\frac{r}{\sqrt{2}}\right) - \frac{r^5}{5}\right)\Big|_0^1$$

$$2\pi k\left(-\frac{1}{4} + \frac{1}{4} + \frac{1}{2}\frac{\pi}{4} - \frac{1}{5}\right) = 2\pi k\left(\frac{\pi}{8} - \frac{1}{5}\right)$$