FUNCIONES DEFINIDAS FORMA IMPLÍCITA

* **Ejercicio 22** Demostrar $z = f(x,y)$ en entorno $E(x_0 y_0)$ donde práct para $A = (x_0\, y_0\, z_0)$. Hallar $\nabla f(x_0 y_0)$

ⓐ $x^2 - y^2 + z^2 = 0$, $A = (4, 5, z_0)$. $z_0 > 0$

$F(x,y,z) = x^2 - y^2 + z^2$ ↪ $\text{Dom } F = \{(x,y,z) \in \mathbb{R}^3 / z > 0\}$

↳ Ver condiciones de teorema

Ⓘ $G(4,5,z_0) = 4^2 - 5^2 + z_0^2 = -9 + z_0^2 = 0$ → para $z_0 = 3$ el punto $A = (4,5,3) \in \text{Dom } F$

Ⓘⓘ G es de clase C^1 en entorno de $E(A)$ → es función polinómica continua y derivable en su dominio

Ⓘⓘⓘ $\frac{\partial G}{\partial z}(4,5,3) = 2z \Big|_{(4,5,3)} = 2.3 = 6 \neq 0$ → existe funcion implícita $z = f(x,y)$ en entorno $E(4,5)$

⇒ Hallar $\nabla f(x_0\, y_0)$ ⟶ Hallar $\nabla f(4,5) = \left(f_x(4,5) \quad f_y(4,5) \right)$

↳ Hallar $f_x(x,y)$, sabiendo $z = f(x_0, y_0) = 3$.

$$\frac{\partial}{\partial x}\left(x^2 - y^2 + f^2(x,y) \right) = \frac{\partial}{\partial x}(0) \Rightarrow 2x + 2f_x(x,y) f(x,y) = 0 \Rightarrow f_x(x,y) = \frac{-x}{f(x,y)} \Rightarrow \boxed{f_x(4,5) = -\frac{4}{3}}$$

$$\frac{\partial}{\partial y}\left(x^2 - y^2 + f^2(x,y) \right) = \frac{\partial}{\partial y}(0) \Rightarrow -2y + 2f_y(x,y) f(x,y) \Rightarrow f_y(x,y) = \frac{y}{f(x,y)} \Rightarrow \boxed{f_y(4,5) = \frac{5}{3}}$$

ⓑ $z = 2 - \ln(z + 3x - y^2)$, $A = (1,2,2)$

$G(x,y,z) = z - 2 + \ln(z + 3x - y^2)$ $\quad \text{Dom } G = \{(x,y,z) \in \mathbb{R}^3 / z + 3x > y^2\}$ → $A \in \text{Dom } G$

• Ver teorema Función Implícita

Ⓘ $G(1,2,2) = 2 - 2 + \ln(2 + 3 - 4) = 0$

Ⓘⓘ G es clase C^1 función polinómica y logarítmica clase C^1 en su dominio

Ⓘⓘⓘ $\frac{\partial G}{\partial z}(1,2,2) = 1 + \frac{1}{z + 3x - y^2}\Big|_{(1,2,2)} = 1 + \frac{1}{2+3-4} = 2 \neq 0$ $\quad \exists f(x,y) / z = f(x,y)$ en $(x,y) \in E(1,2)$

⇒ Hallar $\nabla f(1,2) = \left(f_x(1,2) \quad f_y(1,2) \right)$

$$\frac{\partial}{\partial x}\Big(\underset{f(x,y)}{z} - 2 + \ln(z + 3x - y^2) \Big) = \frac{\partial}{\partial x}(0) \Rightarrow f_x(x,y) + \underbrace{\frac{1}{f(x,y) + 3x - y}}_{1} \cdot \left(f_x(x,y) + 3 \right) = 0$$

si $z = f(1,2) = 2$

$$\boxed{f_x = \frac{-3}{2}}$$

$$\frac{\partial}{\partial y}\left(f(x,y) - 2 + \ln(f(x,y) + 3x - y^2) \right) = \frac{\partial}{\partial x}(0)$$

⇓

$$f_y(x,y) + \underbrace{\frac{1}{f(x,y) + 3x - y^2}}_{1} \left(f_y(x,y) - 2y \right) = 0$$

$$f_y(1,2) = \frac{4}{2} = 2$$

$$\nabla f(1,2) = \left(-\frac{3}{2}, 2 \right)$$

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* **Ejercicio 23** Demostrar $(x^2+\ln(x+z)-y,\ yz+e^{xz}-1)=(0,0)$ define curva reg en entorno $\underbrace{(1,1,0)}_{A}$

↳ Hallar ec. plano normal a C en dicho punto

↳ Si representa una curva, entonces debería poder ser manejado por una sola variable

$\Rightarrow$ Hallar una función $\rightarrow G: D\subseteq\mathbb{R}^3\rightarrow\mathbb{R}^2 \ /\ G(x,y,z)=(x^2+\ln(x+z)-y,\ yz+e^{xz}-1)$

Dominio G $=\{(x,y,z)\in\mathbb{R}^3 /\ x+z>0\}$ $\rightarrow$ punto $A=(1,1,0)\in$ Dominio G

↳ Al mismo tiempo podemos hallar $G_1:\mathbb{R}^3\rightarrow\mathbb{R} / G_1(x,y,z)=x^2+\ln(x+z)-y$, $G_2:\mathbb{R}^3\rightarrow\mathbb{R}/G_2(x,y,z)=yz+e^{xz}-1$

↳ podríamos intentar poner G_1 y G_2 el valor z en función de $x\,y$ en entorno $(1,1,0)$

- Verificar Teorema Función implícita en G_1

I) $G_1(1,1,0)=1^2+\ln(1+0)-1=0$

II) $G_1\in C^1(E(1,1,0))$ ya que es logarítmica y polinómica en su dom

III) $\dfrac{\partial G_1}{\partial z}(1,1,0)=\left.\dfrac{1}{x+z}\right|_{(1,1,0)}=1\neq 0$ ↷ Existe función $f(x,y)$ en $E(1,1)$ donde $z=f(x,y)$

- Verificar Teorema en G_2

I) $G_2(1,1,0)=1.0+e^0-1=0$

II) $G_2\in C^1(E(1,1,0))$ ya que es función polinómica y exponencial

III) $\dfrac{\partial G_2}{\partial z}(1,1,0)=\left.y+xe^{xz}\right|_{(1,1,0)}=1+1.e^0=2\neq 0\rightarrow$ existe f_2 en $E(1,1)$ / $z=f(x,y)$

REHACER ATRÁS

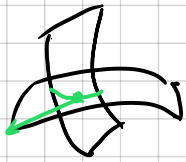
* Ejercicio 23 Demostrar $(x^2+\ln(x+z)-y,\ yz+e^{xz}-1)=(0,0)$ define curva reg en entorno $\underbrace{(1,1,0)}_{A}$

↳ Hallar ec. plano normal a C en dicho punto

podría entender esto como la intersección de dos superficies G_1 y G_2

⇒ Definir sistema de ecuaciones

$$\begin{cases} x^2+\ln(x+z)-y=0 \\ yz+e^{xz}-1=0 \end{cases}$$



$$\boxed{C=G_1\cap G_2 \text{ en } E(1,1,0)}$$

- Verificar Teorema Función implícita en G_1

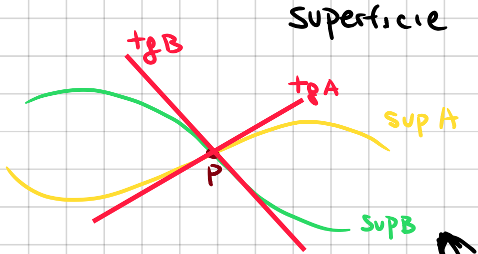
(I) $G_1(1,1,0)=1^2+\ln(1+0)-1=0$

(II) $G_1\in C^1(E(1,1,0))$ ya que es logarítmica y polinómica en su dom

(III) $\dfrac{\partial G_1}{\partial z}(1,1,0)=\dfrac{1}{x+z}\Big|_{(1,1,0)}=1\neq 0$ → Existe función $f(x,y)$ en $E(1,1)$ donde $\underbrace{z=f(x,y)}_{\text{superficie}}$

$\vec{F}(x,y)=(x,y,f(x,y))$

Fig análisis
Int. superficies
Corte con plano



- Verificar Teorema en G_2

(I) $G_2(1,1,0)=1.0+e^0-1=0$

(II) $G_2\in C^1(E(1,1,0))$ ya que es función polinómica y exponencial

(III) $\dfrac{\partial G_2}{\partial z}(1,1,0)=y+xe^{xz}\Big|_{(1,1,0)}=1+1.e^0=2\neq 0$ → existe h en $E(1,1)$ / $z=h(x,y)$ → superficie

$\vec{H}(x,y)=(x,y,h(x,y))$

- Armar plano normal a C en $(1,1,0)$

si una curva es int de dos sup., su tg en ese punto = intersección de los planos tangentes a ese punto

Hallar normales de los planos tg en $\vec{F}_1$ y $\vec{F}_2$

la intersección de los planos tg son producto vectorial de vectores normales de los planos

- Hallar normal en $\vec{F}_1(x,y)$

$$\frac{\partial F}{\partial x}(1,1)\times\frac{\partial F}{\partial y}(1,1)=\vec{N}_1(1,1)=\left(-f_x(1,1),-f_y(1,1),1\right)$$

↳ Hallar $f_x(1,1)$

$$\frac{\partial}{\partial x}\left(x^2+\ln(x+z)-y\right)=\frac{\partial}{\partial x}(0)$$

$$2x+\frac{1}{x+f(x,y)}\cdot\left(1+f_x(x,y)\right)=0$$

evaluar $(x,y)=(1,1)$

$$2+\frac{1}{1+0}\left(1+f_x(1,1)\right)=0$$

$$f_x(1,1)=-3$$

↳ Hallar $f_y(1,1)$

$$\frac{\partial}{\partial y}\left(x^2+\ln(x+f(x,y))-y\right)=\frac{\partial}{\partial y}(0)$$

$$\frac{1}{x+f(x,y)}\cdot f_y(x,y)-1=0$$

evaluar $(x,y)=(1,1)$

$$f_y(1,1)=1$$

$$\vec{N}_1=(3,-1,1)$$

⇒ continuación ej 23

- Hallar normal en $\vec{H}(x,y)$

$$\vec{N}_z = \frac{\partial H}{\partial x} \times \frac{\partial H}{\partial y} = (-h_x, -h_y, 1)$$

↳ Hallar h_x

$$\frac{\partial}{\partial x}\left(yz + e^{xz} - 1\right) = \frac{\partial}{\partial x}(0)$$

evaluar en (1,1)

$$y\,h_x(x,y) + \underbrace{e^{xh(x,y)}}_{0} \cdot \left(h(x,y) + h_x(x,y)\,x\right) = 0$$

$$h_x(1,1) + e^0 \cdot \left(\underbrace{h(1,1)}_{0} + h_x(1,1) \cdot 1\right) = 0$$

$$= \quad 2h_x(1,1) = 0 \Rightarrow \boxed{h_x(1,1) = 0}$$

↳ Hallar h_y

$$\frac{\partial}{\partial y}\left(y\,h(x,y) + e^{xh(x,y)} - 1\right) = \frac{\partial}{\partial y}(0)$$

eval (1,1)

$$h(x,y) + h_y(x,y) \cdot y + e^{xh(x,y)} \cdot \left(x\,h_y(x,y)\right) = 0$$

$$\underbrace{h(1,1)}_{0} + h_y(1,1) + e^0 \cdot \left(h_y(1,1)\right) = 0$$

$$\boxed{h_y(1,1) = 0}$$

$$\vec{N}_z = (0,0,1)$$

- Hallar vector tg a curva

$$\vec{V} = \vec{N}_1 \times \vec{N}_z$$

$$\vec{V} = \begin{vmatrix} i & j & k \\ 3 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = (-1, -3, 0)$$

- Plano normal a curva c

$$\vec{v} \cdot (x - A) = 0$$

$$(-3, -1, 0)\,(x-1, y-1, z) = 0$$

$$-3x - y + 4 = 0$$

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* Ejercicio 25

$$\begin{array}{ll} F(\vec{x}) & xy^2 + z\mu + v^2 = 3 \\ G(\vec{x}) & x^3 z + 2y - \mu v = 2 \\ H(\vec{x}) & x\mu + yv - xyz = 1 \end{array}$$

Demostrar satisface $(x_0, y_0, z_0, \mu_0, v_0) = (1,1,1,1,1)$

define x, y, z en función de (μ, v) en entorno $(\mu_0, v_0) = (1,1)$

Calcular $y'_\mu(\mu_0\ v_0)$

↳ Demostrar que cuando $(\mu_0, v_0) = (1,1)$, se def $x(\mu,v)$, $y(\mu,v)$, $z(\mu,v)$ en $(1,1)$

↳ 3 ecuaciones ⟶ 5 incógnitas ⟶ no puedo demostrar $x(\mu,v)$ directamente pero sí $\mu(x,y,z)$

$F(\vec{x}) = xy^2 + z\mu + v^2 - 3 \qquad G(\vec{x}) = x^3 z + 2y - \mu v - 2 \qquad H(\vec{x}) = x\mu + yv - xyz - 1$

• Teorema Función Implícita ⟶ demostrar $x(\mu,v)$ $y(\mu,v)$ $z(\mu,v)$

$F, G, H \in (C^1(\mathbb{R}^5))$

(I) $F(A) = 0 \quad G(A) = 0 \quad H(A) = 0$

(II) $\nabla F(\vec{x}) = (y^2, 2xy, \mu, z, 2v) \quad \nabla G(\vec{x}) = (3zx^2, 2, x^3, -v, -\mu) \quad \nabla H(\vec{x}) = (\mu - yz, v - xz, xy, x, y)$

(III) $$\frac{\partial(F,G,H)}{\partial(x,y,z)}(A) = \begin{vmatrix} F'_x & F'_y & F'_z \\ G'_x & G'_y & G'_z \\ H'_x & H'_y & H'_z \end{vmatrix}_{(A)} = \begin{vmatrix} y^2 & 2xy & \mu \\ 3zx^2 & 2 & x^3 \\ \mu+yz & v+xz & xy \end{vmatrix}_{(A)} = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 0 & 0 & -1 \end{vmatrix}$$

que los coeficientes no sean nulos

Laplace F_3 $= (-1)^{3+3}(-1)\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = -(2-6) = 4 \neq 0$

Existen $x(\mu,v)$ $y(\mu,v)$ $z(\mu,v)$ en un entorno $(\mu,v) \in E(1,1)$

→ Hallar $y'_\mu(1,1)$

$$\begin{cases} \frac{\partial}{\partial\mu}(xy^2 + z\mu + v^2) = \frac{\partial}{\partial\mu}(3) \\ \frac{\partial}{\partial\mu}(x^3 z + 2y - \mu \cdot v) = \frac{\partial}{\partial\mu}(2) \\ \frac{\partial}{\partial\mu}(x\mu + yv - xyz) = \frac{\partial}{\partial\mu}(1) \end{cases}$$

$$\begin{cases} x_\mu\, y(\mu,v) + 2y(\mu,v)\, y_\mu\, x(\mu,v) + z_\mu \cdot \mu + z(\mu,v) = 0 \\ 3x^2 x_\mu z + z_\mu x^3 + 2y_\mu - v = 0 \\ x_\mu \cdot \mu + x + v\, y_\mu - \Big(x_\mu yz + \underbrace{\tfrac{\partial}{\partial\mu}(yz)}_{y_\mu z + z_\mu y}\, x\Big) = 0 \end{cases}$$ evaluar $(\mu,v) = (1,1)$

$$\boxed{y'_\mu(1,1) = -\frac{3}{2}}$$

$$\begin{cases} x_\mu + 2y_\mu + z_\mu = -1 \\ 3x_\mu + 2y_\mu + 3z_\mu = 1 \\ x_\mu + 1 + y_\mu - (x_\mu + y_\mu + z_\mu) = 0 \end{cases}$$

$$\begin{cases} x_\mu + 2y_\mu + z_\mu = -1 \to x_\mu + 2y_\mu = -2 \to x_\mu = -2 - 2y_\mu \\ 3x_\mu + 2y_\mu + z_\mu = 1 \to 3x_\mu + 2y_\mu + 1 = 1 \\ z_\mu = 1 \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & 2 & -2 \\ 3 & 2 & 0 \end{array}\right) \xrightarrow{F_2 - F_1} \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 2 & 0 & 2 \end{array}\right)$$

$$\boxed{y_\mu = \frac{-3}{2}} \qquad \boxed{x_\mu = 1}$$

* Ejercicio 26 $\boxed{pV = T - \frac{4p}{T^2}}$ $(p_0, V_0, T_0) = (1, 1, 2)$

(a) Hallar $\frac{\partial T}{\partial p}$ y $\frac{\partial T}{\partial V}$ en (p_0, V_0) → tengo que hallar $T(p, v)$ osea T en función (p, V)

$F(p, v, T) = T - \frac{4p}{T^2} - pV$ → Dom f : $(T \neq 0)$

- Probar teorema función implícita

(I) $F(1, 1, 2) = 2 - \frac{4 \cdot 1}{2^2} - 1 \cdot 1 = 0$ — Grado C^1 en su dominio y $T \neq 0$

(II) $\nabla F(p, v, T) = \left(\frac{-4}{T^2} - V, -p, 1 - 4p \cdot (-2) \frac{1}{T^3}\right)$ gradiente continua en su dominio $\vec{F}$

(III) $\nabla F(1,1,2) = (-2, -1, 1) \neq 0$ → existe T en función de p y V → $T = T(p, V)$ definida en $(1,1)$

- Hallar $\frac{\partial T}{\partial p}(1,1)$

$$\frac{\partial}{\partial p}\left(T - \frac{4p}{T^2} - pV\right) = \frac{\partial}{\partial p}(0)$$

$$\frac{\partial T}{\partial p} - \frac{4 \cdot T^2 - 2T \cdot \frac{\partial T}{\partial p} 4p}{T^4} - V = 0$$

eval $(1,1)$

$$\frac{\partial T}{\partial p} - \frac{4 \cdot 2^2 - 2^2 \frac{\partial T}{\partial p} 4}{2^4} - 1 = 0$$

$$\frac{\partial T}{\partial p} - \left(1 - \frac{\partial T}{\partial p}\right) - 1 = 0$$

$$2\frac{\partial T}{\partial p} = 2 \rightarrow \boxed{\frac{\partial T}{\partial p} = 1}$$

- Hallar $\frac{\partial T}{\partial V}(1,1)$

$$\frac{\partial}{\partial V}\left(T - \frac{4p}{T^2} - pV\right) = \frac{\partial}{\partial V}(0)$$

$$\frac{\partial T}{\partial V} - \frac{4p \cdot (-2)}{T^3} \cdot \frac{\partial T}{\partial V} - p = 0$$

eval $(1,1,2)$

$$\frac{\partial T}{\partial V} + \frac{\partial T}{\partial V} - 1 = 0 \rightarrow \boxed{\frac{\partial T}{\partial V} = \frac{1}{2}}$$

(b) Hallar $T \neq 2$ que satisfaga $p = 1$ y $V = 1$

$$p \cdot V = T - \frac{4p}{T^2} \rightarrow 1 \cdot 1 = T - \frac{4}{T^2} \rightarrow T - \frac{4}{T^2} - 1 = 0 \rightarrow T^2\left(T - \frac{4}{T^2} - 1\right) = 0$$

WTF

$$T^3 - 4 - T^2 = 0$$

En un entorno $(p, V) = (1, 1)$ se define función T de clase C^1 (continua y derivable) $T(p, V) = 2$

→ que es inyectiva por lo que no hay otro valor de T

(c) Acotar error ΔT sabiendo $p = 1 \pm 0{,}001$ y $V = 1 \pm 0{,}002$ en $T = 2 + \Delta T$

↳ Hallar error mediante aproximación lineal?

$$T(1,1) = 2$$

$$L(1,1) = T(1,1) + \frac{\partial T}{\partial p}(1,1)\Delta p + \frac{\partial T}{\partial V}(1,1)\Delta V$$

$$L(1,1) = 2 + 1 \cdot 0{,}001 + \frac{1}{2} \cdot 0{,}002 = \underset{\downarrow \, T}{2} + \underbrace{0{,}002}_{\Delta T_{max} = 0{,}002}$$

* Ejercicio 27 Hallar cartesiana para tg C

(a) $C = \{(x,y) \in \mathbb{R}^2 / x^2 + y^2 + 2xy = 4\}$ en $(1,1)$ $F(x,y) = x^2+y^2+2xy-4$. en $(x,y)=1$

- Ver si puedo hallar $y / y = f(x)$ en $x_0 = 1$

(I) $F(1,1) = 1^2 + 1^2 + 2.1.1 - 4 = 2+2-4 = 0$

(II) F es clase C^1 en su dominio $\text{Dom } F = \mathbb{R}^2$ ya que es polinómica

(III) $\frac{\partial F}{\partial y}(1,1) = 2x + 2y = 4 \neq 0 \rightarrow$ existe $f / y = f(x)$ en $x_0 \in E(1)$

→ Hallar recta tg

↳ Hallar un vector ortogonal al vector tg en $(1,1) \rightarrow \nabla F(1,1) = \left(\frac{\partial F}{\partial x}(1,1), \frac{\partial F}{\partial y}(1,1)\right)$

$\nabla F(x,y) = (2x+2y, 2y+2x)$ $\nabla F(1,1) = (4,4)$

↳ Hallar un vector tangente a C en $(1,1) \rightarrow$ un vect $\vec{v} \perp (4,4)$

Un vector tg es $\vec{v} = (4,-4)$ → recta tg $= \lambda(4,-4) + (1,1)$

(b) $C = \{(x,y) \in \mathbb{R}^2 / xy + \ln(y) = e^x\}$ en $(0,e)$ $F(x,y) = xy + \ln(y) - e^x$ → $\text{Dom } F = \{(x,y) / y > 0\}$

(I) $F(0,e) = 0.e + \ln(e) - e^0 = 1 - 1 = 0$

(II) $\nabla F(x,y) = \left(y - e^x, x + \frac{1}{y}\right) \rightarrow$ F clase C^1 ya que sus derivadas parciales lo son

(III) $\frac{\partial F}{\partial y}(0,e) = 0 + \frac{1}{e} \neq 0$ → $\exists f(x) / y = f(x)$ en $x_0 \in E(0)$

→ Hallar recta tg → $\nabla F(0,e) = \left(e-1, 0 + \frac{1}{e}\right) = \left(e-1, \frac{1}{e}\right)$

recta tg $\vec{r}(\lambda) = \lambda\left(-\frac{1}{e}, e-1\right) + (0,e)$, $\lambda \in \mathbb{R}$

(c) $C = \{(x,y,z) \in \mathbb{R}^3 / (xy+z, y+x-2) = (3,5)\}$ en $(1,6,-3)$ $\begin{cases} F(x,y,z) = xy+z-3 \\ G(x,y,z) = y+x-2-5 \end{cases}$

(I) $F(1,6,-3) = 1.6 - 3 - 3 = 0$ $G(1,6,-3) = 1+6-2-5 = 0$

(II) $\nabla F(x,y,z) = (y, x, 1)$ $\nabla G(x,y,z) = (1,1,0)$ → F y G $\in C^1(E(1,6,-3))$

(III) $\frac{\partial(F,G)}{\partial(y,z)}(1,6,-3) = \begin{vmatrix} x & 1 \\ 1 & 0 \end{vmatrix}_{(1,6,-3)} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0$ → existe $z = g(x)$, $y = f(x)$ en $x_0 \in E(1)$

Curva $\vec{h}(x) = (x, y = f(x), g(x))$ $f'(1) = y_x(1) = -1$ $g'(1) = z_x(1) = 5$

$\vec{h}'(x) = (1, \underbrace{f'(x)}, \underbrace{g'(x)})$ → $h'(1) = (1,-1,5)$

→ recta tg $\vec{r}(\lambda) = \lambda(1,-1,5) + (1,6,-3)$

$\begin{cases} \frac{\partial}{\partial x}(xy - z) = \frac{\partial}{\partial x}(3) \\ \frac{\partial}{\partial x}(y+x-2) = \frac{\partial}{\partial x}(5) \end{cases} \rightarrow \begin{cases} y + y_x . x - z_x = 0 \\ y_x + 1 = 0 \end{cases}$

$y + y_x . x - z_x = 0 \rightarrow 6 + y_x - z_x = 0 \rightarrow \boxed{z_x = 6 + (-1) = 5}$

$y_x + 1 = 0 \rightarrow \boxed{y_x = -1}$

* $\underline{\text{Ejercicio 29}}$: Curva "C" $y=x^2$ int $e^{xz-1}-xy+\ln(yz)=0$

Hallar dist rect tg en $A=(1,1,1)$ a $x+y=8$

↳ Hallar intersección

$F(x,y,z) = y-x^2 \quad G(x,y,z) = e^{xz-1}-xy+\ln(yz)$

no hace falta probar que define curva porque lo dice en enunciado

↳ Hallar $\nabla F(1,1,1)$ y $G(1,1,1)$

$\nabla F(x,y,z) = (-2x,\ 1,\ 0) \qquad \nabla G(x,y,z) = \left(e^{xz-1}\cdot z - y\,\underbrace{\ln(yz)}_{0},\ -x+\frac{1}{yz}\cdot z,\ xe^{xz-1}+\frac{1}{yz}y \right)$

$\nabla F(1,1,1) = (-2,1,0) \qquad \nabla G(1,1,1) = \left(1\cdot e^0 - 1,\ -1+\frac{1}{1},\ 1e^0+\frac{1}{1}\cdot\right)$

$\nabla G(1,1,1) = (0,\ 0,\ 1)$

↳ Hallar vector tg a curva en A

$$\vec{v} = \nabla F(A) \times \nabla G(A) = \begin{vmatrix} i & j & k \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (1,\ 2,\ 0)$$

- recta tang a curva en A $\vec{r}(\lambda) = \lambda(1,2,0)+(1,1,1)$, $\lambda \in \mathbb{R}$

→ Hallar intersección con plano $x+y=8$ $\quad \vec{r}(\lambda) = (\lambda+1,\ 2\lambda+1,\ 1)$

$(\lambda+1)+(2\lambda+1)+0\cdot 1 = 8 \rightarrow 3\lambda = 8-2 \rightarrow \boxed{\lambda = \frac{6}{3} = 2} \qquad \boxed{\vec{r}(2) = (3,5,1)}$

intersección pto $(3,5,1)$

Hallar distancia entre A y $\vec{r}(2)$

$\sqrt{A-\vec{r}(2)} = \sqrt{(1,1,1)-(3,5,1)} = \sqrt{2^2+4^2+0^2} = \sqrt{20} = 2\sqrt{5}$

* Ejercicio 30 : $\begin{cases} \ln(x-z) + y + x = 5 \\ e^{yz} + z - x = 0 \end{cases}$ Define curva C en $A = (5,0,4)$. Hallar int recta tg de curva en A con el $x+z=10$

$\begin{cases} F(x,y,z) = \ln(x-z) + y + x - 5 \\ G(x,y,z) = e^{yz} + z - x \end{cases}$ intersección de superficies

$\frac{1}{x-z} \frac{\partial}{\partial z}(-z) = -\frac{1}{x-z}$

• Hallar $\nabla F(A)$ y $\nabla G(A)$

$\nabla F(x,y,z) = \left(\frac{1}{x-z} + 1, \ 1, \ \frac{1}{x-z} \right)$

$\nabla F(5,0,4) = \left(\frac{1}{5-4} + 1, \ 1, \ \frac{1}{5-4} \right)$

$\nabla F(A) = (2, 1, 1)$

$\nabla G(x,y,z) = \left(-1, \ z e^{yz}, \ y e^{yz} + 1 \right)$

$\nabla G(5,0,4) = \left(-1, \ 4e^0, \ 0e^0 + 1 \right)$

$\nabla G(A) = (-1, 4, 1)$

• Hallar vector tg a curva

$\vec{v} = \nabla F(A) \times \nabla G(A) = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ -1 & 4 & 1 \end{vmatrix} = (-3, -3, 9)$

• recta tangente a C en A

$\vec{r}(\lambda) = \lambda(1,1,-3) + \overbrace{(5,0,4)}^{A}$

$= (\lambda+5, \ \lambda, \ 4-3\lambda)$

• Hallar intersección de recta tg con plano $x+z=10$

$(\lambda+5) + 0\lambda + (4-3\lambda) = 10$

$-2\lambda = 10 - 9$

$\boxed{\lambda = -\frac{1}{2}}$

$\boxed{r\left(-\frac{1}{2}\right) = \left(\frac{9}{2}, -\frac{1}{2}, \frac{11}{2}\right)}$ intersección recta tg a C en A con plano $x+z=10$