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COMPOSICIÓN DE FUNCIONES

* Ejercicio 1   Calcular matriz jacobiana $h = f \circ g$ en punto indicado

ⓐ $f(\mu, v) = \mu v^2 + 2\mu v$, $\vec{g}(x,y) = (x\sqrt{y+1},\ y-x)$, $A = (2,3)$ $g(A) = g(2,3) = (4, 1)$

- Hallar $Dg(A)$

$$Dg(x,y) = \begin{pmatrix} \sqrt{y+1} & \frac{x}{2\sqrt{y+1}} \\ -1 & 1 \end{pmatrix} \qquad Dg(A) = Dg(2,3) = \begin{pmatrix} 2 & \frac{1}{2} \\ -1 & 1 \end{pmatrix}$$

- Hallar $Df(g(A))$

$$Df(\mu,v) = \begin{pmatrix} v^2+2v & 2\mu v + 2\mu \end{pmatrix} \qquad Df(g(A)) = Df(4,1) = \begin{pmatrix} 3 & 16 \end{pmatrix}$$

- Hallar $D_{f\circ g}(A)$

$$Dh(A) = D_{f\circ g}(A) = \begin{pmatrix} 3 & 16 \end{pmatrix} \begin{pmatrix} 2 & 1/2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -10 & \frac{35}{2} \end{pmatrix}$$

ⓑ $\vec{f}(\mu,v) = (\mu v,\ \mu + v^2,\ \ln(v))$, $\vec{g}(x,y) = (x^2 - y^2,\ xy)$, $A = (1,2)$

- Hallar $Dg(A)$

$$Dg(x,y) = \begin{pmatrix} 2x & -2y \\ y & x \end{pmatrix} \qquad Dg(1,2) = \begin{pmatrix} 2 & -4 \\ 2 & 1 \end{pmatrix}$$

- Hallar $Df(g(A))$, $g(A) = (-3, 2)$

$$Df(\mu,v) = \begin{pmatrix} v & \mu \\ 1 & 2v \\ 0 & \frac{1}{v} \end{pmatrix} \qquad Df(-3,2) = \begin{pmatrix} 2 & -3 \\ 1 & 4 \\ 0 & \frac{1}{2} \end{pmatrix}$$

- Hallar $D_h(A) = D_{f\circ g}(A)$

$$D_{f\circ g}(A) = \underset{3\times 2}{\begin{pmatrix} 2 & -3 \\ 1 & 4 \\ 0 & \frac{1}{2} \end{pmatrix}} \underset{2\times 2}{\begin{pmatrix} 2 & -4 \\ 2 & 1 \end{pmatrix}}$$

$$= \begin{pmatrix} -2 & -11 \\ 10 & 0 \\ 1 & 1/2 \end{pmatrix}$$

ⓒ $f(x,y) = x^3 + y^2$. $\vec{g}(t) = (\mathrm{sen}(t) - 1,\ 2 + t + t^2)$, $t_0 = 0$

- Hallar $Dg(0)$

$$Dg(t) = \begin{pmatrix} \cos(t) \\ 1+2t \end{pmatrix} \qquad Dg(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- Hallar $Df(g(0))$, $g(0) = (-1, 2)$

$$Df(x,y) = \begin{pmatrix} 3x^2 & 2y \end{pmatrix} \qquad Df(-1,2) = \begin{pmatrix} 3 & 4 \end{pmatrix}$$

- Hallar $D_h(0)$

$$D_h = D_{f\circ g}(0) = \underset{1\times 2}{\begin{pmatrix} 3 & 4 \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} = \underset{=1\times 1}{[7]}$$

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* Ejercicio 2 $f(x,y) = \text{sen}((x-2)^2 + y - 1)$, expresar $f(x,y) = g(h(x,y))$, indicando Dominio y codominio

$$f(x,y) = \underbrace{\text{sen}}_{g(t)}\big(\underbrace{(x-2)^2+y-1}_{h(x,y)}\big) \qquad g(t) = \text{sen}(t) \qquad h(x,y) = (x-2)^2 + y - 1$$

$\Rightarrow f: D(h) \subseteq \mathbb{R}^2 \to Im(g) \in [0,1] \;/\; f(x,y) = g(h(x,y))$

↳ Dominio $f = \mathbb{R}$

↳ Codominio $f = f(x,y) \in [0,1]$

• Hallar Dom(h) y Im(h)

$Dom(h) = \mathbb{R}^2$, $Im(h) = \mathbb{R}$

• Dom(g), Im(g)

$Dom(g) = \mathbb{R}$ $Im(g) \in [0,1]$

* Ejercicio 3 = $f: \mathbb{R}^2 \to \mathbb{R}$, $\vec{g}(x,y) = \left(\sqrt{1-x^2-y^2},\ \ln(x^2+4y^2-16)\right)$

¿Es posible definir $h = f \circ g$?

↳ para definir $h = f \circ g$ entonces $\boxed{Im(g) \subseteq \mathbb{R}^2}$ → hay que ver el codominio de $\vec{g}(x,y)$

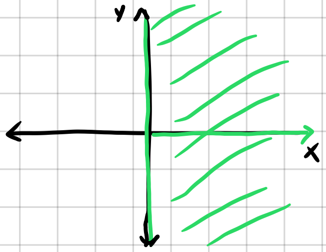
• Reescribir $\vec{g}$ → $\vec{g}(\mu,v) = \left(\sqrt{1-\mu^2-v^2},\ \ln(\mu^2+4v^2-16)\right)$

$X(\mu,v) = \sqrt{1-\mu^2-v^2} \to X(\mu,v) \geq 0$ en dominio de $\vec{g}$] $Im(g)$

$Y(\mu,v) = \ln(\mu^2+4v^2-1) \to Y(\mu,v) \in \mathbb{R}$ → en dom de $\vec{g}$

$Im(g) = \{(x,y) \in \mathbb{R}^2 \,/\, x \geq 0\}$ $\boxed{Im(g) \subseteq D_f}$

↳ si g está def en su dom



• Hallar Dom g

$X(\mu,v) = \sqrt{1-\mu^2-v^2} \to \mu^2+v^2 \leq 1$

$y(\mu,v) = \ln(\mu^2+4v^2-1) \to \mu^2+4v^2-1 > 0 \to \mu^2+4v^2 > 1$

$\Rightarrow \begin{cases} \mu^2+v^2 \leq 1 \\ \mu^2+4v^2 > 1 \end{cases}$ es imposible cumplir los dos condiciones al mismo tiempo ↗ g no está definida en ningún punto de $\mathbb{R}^2$

* Ejercicio 4   Demostrar $f(x,y) = \dfrac{4x^4 + 12x^2y^2 + 9y^4}{4 - 2x^2 - 3y^2}$ es constante en puntos de elipse $2x^2 + 3y^2 = 1$

(Rehacer adión)

↳ Supongamos que la elipse puede ser descripta como función $e(x,y)$, entonces $f(e(x,y)) = k$

↳ $e(x,y)$ es un conjunto nivel   ↳ $(f(e(x,y)))' = k' = 0$ ↷ las derivadas parciales serían nulas

∘ Simplificar $f(x,y)$

$$\begin{array}{l|l} 4x^4 + 12x^2y^2 + 9y^4 & -2x^2-3y^2+4 \\ \hline -\ 4x^4 + 6x^2y^2 - 8x^2 & -2x^2-3y^2+4 \\ \quad 6x^2y^2 + 9y^4 - 8x^2 & \\ \quad -\ 6x^2y^2 + 9y^4 + 12y^2 & \\ \qquad -8x^2 - 12y^2 & \\ \qquad -\ 8x^2 + 12y^2 + 16 & \\ \qquad\quad 16 & \end{array}$$

$$f(x,y) = \frac{16}{-2x^2-3y^2+4} + (-2x^2-3y^2+4)$$

• Demostrar que $e(x,y)$ es conjunto nivel

$$f(x,y) = \frac{4x^4 + 12x^2y^2 + 9y^4}{4 - 2x^2 - 3y^2} = k$$

$4x^4 + 12x^2y^2 + 9y^4 = k(4 - 2x^2 - 3y^2)$

$k = \frac{1}{4}$ $\begin{cases} 4x^4 + 12x^2y^2 + 9y^4 = 4k - 2kx^2 - 3ky^2 \\ 4x^4 + 12x^2y^2 + 9y^4 = 1 - \frac{1}{2}x^2 - \frac{3}{4}y^2 \end{cases}$

↗ Hallar k para que conjunto nivel sea la misma ec. de elipse

* Ejercicio 4 Demostrar $f(x,y) = \dfrac{4x^4 + 12x^2y^2 + 9y^4}{4 - 2x^2 - 3y^2}$ es constante en puntos de elipse $2x^2 + 3y^2 = 1$

$\frac{\partial}{\partial t} f(e(t)) = 0$ $\boxed{f(e(t)) \in \mathbb{R}}$

- Hallar parametrización de elipse

$$x = \frac{1}{\sqrt{2}}\cos(t) \quad y = \frac{1}{\sqrt{3}}\operatorname{sen}(t) \Rightarrow e(t) = \left(\tfrac{1}{\sqrt{2}}\cos(t), \tfrac{1}{\sqrt{3}}\operatorname{sen}(t)\right), \ t \in [0, 2\pi]$$

- Hallar $f(e(t))$

$$f(e(t)) = f\left(\tfrac{1}{\sqrt{2}}\cos(t), \tfrac{1}{\sqrt{3}}\operatorname{sen}(t)\right)$$

$$= \frac{4\left(\frac{1}{\sqrt{2}}\cos t\right)^4 + 12\left(\frac{1}{\sqrt{2}}\cos t\right)^2\left(\frac{1}{\sqrt{3}}\operatorname{sen} t\right)^2 + 9\left(\frac{1}{\sqrt{3}}\operatorname{sen} t\right)^4}{4 - 2\left(\frac{1}{\sqrt{2}}\cos t\right)^2 - 3\left(\frac{1}{\sqrt{3}}\operatorname{sen} t\right)^2} = \frac{\cos^4 t + 12\left(\frac{1}{2}\cos^2 t\right)\left(\frac{1}{3}\operatorname{sen}^2 t\right) + \operatorname{sen}^4 t}{4 - \cos^2 t - \operatorname{sen}^2 t}$$

$$= \frac{\cos^4 t + 2\cos^2 t \operatorname{sen}^2 t + \operatorname{sen}^4 t}{3} = \frac{(\cos^2 t + \operatorname{sen}^2 t)^2}{3} = \frac{1}{3}$$

en puntos (x,y) de la elipse, $f(x,y) = \frac{1}{3}$ constante

* Ejercicio 5 $f(x,y) = (xy^4 + y^2x^3, \ln(x))$ $\vec{g}(\mu,v) = (v\sqrt{\mu}, \operatorname{sen}(\mu-1)/\mu)$

(a) Hallar dominios naturales y expresiones $\vec{h} = \vec{f} \circ \vec{g}$ y $\vec{w} = \vec{g} \circ \vec{f}$

- Dominios naturales

$$\operatorname{Dom} f = \{(x,y) \in \mathbb{R}^2 / x > 0\} \qquad \operatorname{Dom} g = \{(\mu,v) \in \mathbb{R}^2 / \mu > 0\}$$

- Hallar expresiones

$$\vec{h} = \vec{f} \circ \vec{g} = f(g(\mu,v)) = (f_1(g(\mu,v)), f_2(g(\mu,v)))$$

$$f_1(g(\mu,v)) = v\sqrt{\mu}\left(\frac{\operatorname{sen}(\mu-1)}{\mu}\right)^4 + \left(\frac{\operatorname{sen}(\mu-1)}{\mu}\right)^2(\mu\sqrt{\mu})^3 = v\sqrt{\mu}\left(\frac{\operatorname{sen}(\mu-1)}{\mu}\right)^4 + \left(\frac{\operatorname{sen}(\mu-1)}{\mu}\right)^2 (v\sqrt{\mu})^3$$

$$f_2(g(\mu,v)) = \ln(\mu\sqrt{\mu}) \quad \underline{\mu > 0}$$

$$\vec{h}(\mu,v) = \left(v\sqrt{\mu}\left(\frac{\operatorname{sen}(\mu-1)}{\mu}\right)^4 + \left(\frac{\operatorname{sen}(\mu-1)}{\mu}\right)^2(v\sqrt{\mu})^3, \ \ln(v\sqrt{\mu})\right)$$

$$\vec{h} = D_g \subseteq \mathbb{R}^2 \to \mathbb{R}^2 \longrightarrow D_h = \{(\mu,v) \in \mathbb{R}^2 / \mu > 0, v > 0\}$$

$$\vec{w} = g \circ f = g(f(x,y)) = (g_1(f(x,y)), g_2(f(x,y)))$$

$$g_1(f(x,y)) = \ln(x)\sqrt{xy^2 + y^2x^3}$$

$$g_2(f(x,y)) = \frac{\operatorname{sen}(xy^2 + y^2x^3 - 1)}{xy^2 + y^2x^3}$$

$$D_w = \{(x,y) \in \mathbb{R}^2 / x > 0, \ \underbrace{xy^2 + y^2x^3}_{\geq 0 \ \forall (x,y),\ x>0} > 0\}$$

- hallar $h'_v(1,e)$ aplicando regla de cadena

$$\vec{h}'_v = (1,e) = (h_{1v}'(1,e), h_{2v}'(1,e))$$

$$h'_{1v} \to g_1 \to \mu, v; \quad g_2 \to \mu, v$$

$$= \frac{\partial h_1}{\partial v}(\mu,v) = \frac{\partial h}{\partial g}(g(\mu,v)) \cdot \frac{\partial g}{\partial \mu}(1,e)$$

$(1,e)$

• hallar $h'_v(1,e)$ aplicando regla de cadena y $\vec{h}(u,v) = \left(v\sqrt{u}\left(\frac{\operatorname{sen}(u-1)}{u}\right)^4 + \left(\frac{\operatorname{sen}(u-1)}{u}\right)^2 (v\sqrt{u})^3,\ \ln(v\sqrt{u}) \right)$

$$\frac{\partial h_1}{\partial v}(u,v) = \sqrt{u}\left(\frac{\operatorname{sen}(u-1)}{u}\right)^4 + \left(\frac{\operatorname{sen}(u-1)}{u}\right)^2 3v^2(\sqrt{u})^3 \longrightarrow \boxed{\frac{\partial h_1}{\partial v}(1,e) = 0}$$

$$\frac{\partial h_2}{\partial v}(u,v) = \frac{1}{v\sqrt{u}}\cdot\sqrt{u} = \frac{1}{v} \longrightarrow \boxed{\frac{\partial h_2}{\partial v}(1,e) = \frac{1}{e} = e^{-1}}$$

$$h'_v(1,e) = (0,\ e^{-1})$$

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* Ejercicio 6 $h(u) = f(\vec{g}(u))$. Hallar $h'(0)$, $f(x,y) = x^2+2y$, $\vec{g}(u) = (|u|,\ u^2+u)$

$$h'(0) = D_f(g(0))\cdot g'(0) \rightarrow g(0) = (0,0).$$

• Hallar $D_f \rightarrow \nabla f(x,y)$

$$\nabla f(x,y) = (2x,\ 2) \rightarrow \nabla f(g(0,0)) = (0,2)$$

• Hallar $g'(0)$ $\longrightarrow g = (|u|,\ u^2+u) = \begin{cases} (u,\ u^2+u) & \text{si } u \geq 0 \\ (-u,\ u^2+u) & \text{si } u < 0 \end{cases}$

g no es derivable en $u=0$

no puedo hallar $g'(0)$

no se puede aplicar regla cadena.

• Usar composición $h = f\circ g$

$$f(g(u)) = |u|^2 + 2(u^2+u) = u^2+2u^2+u = 3u^2+2u$$

$$h'(u) = 6u+2 \longrightarrow \boxed{h'(0) = 2}$$

* Ejercicio 7 $h = f\circ\vec{g}$. hallar $\nabla h(A)$

$$\boxed{\underset{(1\times2)}{\nabla h(A)} = \underset{1\times2}{\nabla f(g(A))}\cdot \underset{2\times2}{D_g}}$$

(a) $A = (0,1)$, $f(u,v) = \sqrt{u/v}$, $g(x,y) = (1+\ln(x+y),\ \cos(xy))$. $g(0,1) = (1,1)$

• Hallar $\nabla f(g(A))$

$$\nabla f(u,v) = \left(\frac{1}{2\sqrt{u/v}}\cdot\frac{1}{v},\ \frac{-u}{v^2}\cdot\frac{1}{2\sqrt{u/v}}\right) \rightarrow \nabla f(g(A)) = \nabla f(1,1) = \left(\frac{1}{2},\ -\frac{1}{2}\right)$$

• Hallar $Dg(0,1)$

$$\nabla h'(A) = \left(\frac{1}{2},\ -\frac{1}{2}\right)\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \left(\frac{1}{2},\ \frac{1}{2}\right)$$

$$Dg = \begin{pmatrix} \frac{1}{x+y} & \frac{1}{x+y} \\ -y\operatorname{sen}(xy) & -x\operatorname{sen}(xy) \end{pmatrix} \rightarrow Dg(0,1) = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

(b) $A = (1,0)$ $g(x,y) = (x,\ xe^{y^2},\ x-y)$ $f\in C^1(\mathbb{R}^3)$ $\nabla f(1,1,1) = (3,1,2)$

$$Dg = \begin{pmatrix} 1 & 0 \\ e^{y^2} & xe^{y^2}2y \\ 1 & -1 \end{pmatrix} \quad Dg(1,0) = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & -1 \end{pmatrix} \quad g(1,0) = (1,1,1)$$

$$\nabla h(A) = \nabla f(1,1,1)\cdot Dg(1,0) = \underset{1\times3}{(3,1,2)}\underset{3\times2}{\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & -1 \end{pmatrix}} = (6\ \ -2)$$

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* Ejercicio 8 $\quad \underbrace{z = e^x - x^2y - x}_{h(t)}$ $\begin{cases} x = t-1 \\ y = 2t^2 \end{cases}$ Determinar $f: \mathbb{R}^2 \to \mathbb{R}$ y $\vec{g}\ \mathbb{R} \to \mathbb{R}^2$ / $h = f\circ g$

- Hallar $\vec{g}(t)$

como $g: \mathbb{R} \to \mathbb{R}^2$ entonces $\vec{g}(t) = (x(t), y(t))$ $\quad \vec{g}(t) = (t-1, 2t^2)$

- Hallar $f(t)$

como $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = e^x - x^2y - x$ $\quad \longrightarrow \quad f\circ g = h(t)$

- Demostrar $h(1)$ max relativo $\longrightarrow$ h es una funcion $\mathbb{R} \to \mathbb{R}$ se puede usar derivadas

↳ Hallar $h(t)$

$$h(t) = f\circ g = e^{t-1} - (t-1)^2(2t) - (t-1)$$

→ hallar derivada $h'(t)$

$$h'(t) = e^{t-1} - \left[\frac{d}{dt}\left((t-1)^2\right)2t + \frac{d}{dt}(2t)(t-1)^2\right] - 1$$

$$= e^{t-1} - \left[2(t-1)2t + 2(t-1)^2\right] - 1$$

- Hallar extremos $h'(t) = 0$

$$h(t) = e^{t-1} - \left[2(t-1)2t + 2(t-1)^2\right] - 1 = 0$$

$$t-1 = \ln\left(2(t-1)2t + 2(t-1)^2 - 1\right)$$

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* Ejercicio 9 $\quad$ Sea $w = e^{x-y} - z^2y + x$, $\quad x = v - \mu$, $y = \mu + \mu^3 \ln(v-1)$, $z = \mu v$ $\quad$ Hallar derivada max direccional $w = w(\mu, v)$ en pto $(1,2)$

- Hallar composición de funciones

$f(x,y,z) = e^{x-y} - z^2y + x \qquad g(\mu, v) = \left(v-\mu,\ \mu + \mu^3 \ln(v-1),\ \mu v\right)$

- Hallar $\nabla w(1,2)$

$$\nabla w(1,2) = \nabla f(g(1,2)) \cdot D_g(1,2)$$

- Hallar $\nabla f(g(1,2))$, $\quad g(1,2) = (1, 1, 2)$

$$\nabla f(x,y,z) = \left(e^{x-y} + 1,\ -e^{x-y} - z^2,\ -2yz\right) \qquad \nabla f(1,1,2) = (2, -5, -4)$$

- Hallar $D_g(1,2)$

$$D_g(\mu, v) = \begin{pmatrix} -1 & 1 \\ 1 + 3\mu^2 \ln(v-1) & \mu^3 \frac{1}{v-1} \\ v & \mu \end{pmatrix} \longrightarrow D_g(1,2) = \begin{pmatrix} -1 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$$

- Hallar $\nabla w(1,2)$

$$\nabla w(1,2) = \nabla f(1,1,2) \cdot D_g(1,2) = \underset{1\times 3}{(2, -5, -4)} \underset{3\times 2}{\begin{pmatrix} -1 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}} = (-15 \quad -7)$$

- Hallar $\check{w}_{max}$

$$w_{max} = \frac{\nabla w(1,2)}{\|\nabla w(1,2)\|} = \frac{1}{\sqrt{15^2 - 7^2}}(-15, -7)$$ ↪ max derivada dirección

* Ejercicio 10 Demostrar $z=f(x/y)$ satisface $xz_x+yz_y=0$

↳ $Z = f(x/y)$, podemos decir $z=f(g(u,v))$ donde $f:\mathbb{R}\to\mathbb{R}$ y $g(u,v)=(u/v)$

↳ Podemos ver que xZ_x+yZ_y es $\underbrace{\nabla z \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0}$

→ Hallar ∇z ∘ demostrar $xz_x+yz_y=0$

$$\frac{\partial z}{\partial x}(x,y) = \underbrace{f'(x/y)\cdot\frac{1}{y}}_{z_x'} \qquad \frac{\partial z}{\partial y} = \underbrace{f'(x/y)\cdot\frac{-x}{y^2}}_{z_y}$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$

$$x\, f'(x/y)\frac{1}{y} + \left(-\frac{x}{y^2} f'(x/y)\cdot y\right)$$

$$f'(x/y)\frac{x}{y} - \frac{x}{y}f'(x/y) = 0 \quad ✓$$

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* Ejercicio 11 $f(x,y)=x+y$ $g(u)=(u-1)^2$, verif $\nabla(g\circ f)=0$ en $x+y=1$

∘ $\nabla(g\circ f)(x,y) = g'(x,y)\cdot\nabla f(x,y)$

$$\underset{1\times1}{[2(x+y-1)]}\ \underset{1\times2}{(1 \quad 1)} = \underbrace{[2(x+y-1) \quad 2(x+y-1)]}_{\nabla g\circ f(x,y)}$$

• Hallar parametrización $x+y=1 \to h(x)=(x,1-x)$

• Hallar $\nabla(g\circ f)(x,y)$ con puntos de $x+y=1$

$$\nabla(g\circ f)(h(x)) = \Big(2[(x+1-x)-1] \ .\ 2[(x+1-x)-1]\Big) = (2.0 \quad 2.0) = (0,0) = \vec{0}$$

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* Ejercicio 14 Hallar ec recta tg y normal $x^3+x+y+y^3=2$ en $(0,1)$

• Hallar función $\underbrace{x^3+x+y+y^3}=2$

$G(x,y)=x^3+x+y+y^3-2=0$

∘ ver si existe función implícita $G(x,y)$ en $E(0,1)$

↳ $G(1,0)=1^2+1+0+0^3-2=0$

→ $G\in C^1(E(1,0))$ G es func polinóm.

↳ $\frac{\partial G}{\partial y}(1,0) = 1+3y^2\Big|_{(0,1)} = 4 \neq 0$ existe función f def "parametrización" donde $y=f(x)$ en $E(0,1)$

⇒ Hallar recta tg

$$f'(1) = \frac{-\left(\frac{\partial G}{\partial x}\right)}{\left(\frac{\partial G}{\partial y}\right)}\Bigg|_{(0,1)} = -\frac{1+3x^2}{1+3y^2}\Bigg|_{(0,1)} = \frac{-1}{4}$$

rect tg $y-y_0=m(x-x_0) \to y-1=\frac{-1}{4}(x-0) \to y=\frac{-1}{4}x+1$

* Ejercicio 15 $f: \mathbb{R}^2 \to \mathbb{R}^2 / \vec{f}(u,v) = (x(u,v), y(u,v))$ biyectiva y C^2

$$D_f(1,-2) = \underbrace{\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}}_{2\times 2} \qquad \vec{f}(1,-2) = (1,2) \nearrow \text{punto } \underbrace{(1,2)}_{\mathbb{R}^2} = \underbrace{f(1,-2)}_{u=1,\, v=-2}$$

(a) Hallar vect tg en $(1,2)$ a la curva $\operatorname{Im} f$ por ec $u^2+v^2=5$ $\quad g(t) = (\sqrt{5}\cos(t), \sqrt{5}\operatorname{sen}(t))$

Hallar tangente a circunferencia en $(u,v) = (1,-2)$

Hallar t para $g(t) = (1,-2)$

$$\begin{cases} \sqrt{5}\cos(t) = 1 \\ \sqrt{5}\operatorname{sen}(t) = 2 \end{cases} \longrightarrow t = \operatorname{arcsen}\left(\frac{-2}{\sqrt{5}}\right)$$

$\vec{v} = (-2,1)$ vect tg a circunf

$$g'(t) = (-\sqrt{5}\operatorname{sen}(t), \sqrt{5}\cos(t)) \longrightarrow g'(\arccos(1/\sqrt{5})) = (2, 1)$$

Hallar TL de vector tg en f

$$D_f \cdot \vec{v} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

(b) Hallar vect tg en $(1,-2)$ de preimagen por f de $y = 2x$

↳ Hallar parametrizacion $g(t) = (t, 2t)$

$g'(t) =$

No entendí ni verg que mal redactado esta mierd.

* Ejercicio 1b $\vec{F}(\mu,v) = (\mu\cos(v), \mu\,\text{sen}(v), \mu)$. $(\mu,v)\in\mathbb{R}^2$ y $C: v=\mu^2-1$

ⓐ Hallar parametrizac. reg $C^* \to$ Img C a través de $\vec{F}$ $\;\to\; C: (t, t^2+1)$

$$\begin{cases}\mu = t\\ v = t^2+1\end{cases} \qquad \vec{F}(t,t^2+1) = (t\cos(t^2+1), t\,\text{sen}(t^2+1), t) \to \vec{g}(t)$$

ⓑ probar plano tg S en A contiene recta tg a C^*

- Hallar rect tg a A en $\vec{g}(t)$, donde $A=(a,b,c)$ y $\boxed{g(t)=A}$ un pto cualquiera

$$\vec{g}\,'(t) = (\cos(t^2+1) - 2t^2\text{sen}(t^2+1),\ \text{sen}(t^2+1)+2t^2\cos(t^2+1),\ 1)$$

$$\text{rect tg} = \lambda(\cos(t^2+1)-2t^2\text{sen}(t^2+1), \text{sen}(t^2+1)+2t^2\cos(t^2-1), 1) + \underbrace{(t\cos(t^2+1), t\,\text{sen}(t^2+1), t)}_{A=g(t)}$$

- Hallar plano tg a $\vec{F}(\mu,v)$

↳ Hallar normal

$$N(\mu,v) = \frac{\partial F}{\partial \mu}\times\frac{\partial F}{\partial v} = \begin{vmatrix} i & j & k\\ \cos(v) & \text{sen}(v) & 1\\ -\mu\,\text{sen}(v) & \mu\cos(v) & 0\end{vmatrix} = (-\mu\cos(v), -\mu\,\text{sen}(v), \mu\cos^2(v)+\mu\,\text{sen}^2(v))$$

$$(-\mu\cos(v), -\mu\,\text{sen}(v), \mu)$$

- Hallar plano tg a $\vec{F}(\mu,v)$ en $(\mu,v)\in\mathbb{R}^2$ en $\boxed{A=g(t)}$

$$\text{plano} = N(\mu,v)\cdot(X - A) = 0$$

$$(-\mu\cos(v), -\mu\,\text{sen}(v), \mu)\left[(x,y,z) - A\right] = 0$$

- Probar $N(x-A)=0 \to$ si cumple esto entonces recta tg en ec

$$(-\mu\cos(v), -\mu\,\text{sen}(v), \mu)\left(\left[\lambda g'(t) + \underbrace{g(t)}_{A}\right] - A\right) = 0$$

$$N\cdot g'(t) = \underbrace{(-\mu\cos(v), -\mu\,\text{sen}(v), \mu)}(\cos(t^2+1)-2t^2\text{sen}(t^2+1), \text{sen}(t^2+1)+2t^2\cos(t^2-1), 1)$$

Normal en cualquier punto
de sup. $F(\mu,v)$
como $\vec{F}(\mu,v) = \vec{g}\,'(t)$
entonces $(\mu,v) = (t, t^2+1)$

Demostrar

$$(-t\cos(t^2+1), -t\,\text{sen}(t^2+1), t)(\cos(t^2+1)-2t^2\text{sen}(t^2+1), \text{sen}(t^2+1)+2t^2\cos(t^2+1), 1) = 0$$

$$-t\cos^2(t^2+1) + 2t^3\cos(t^2+1)\text{sen}(t^2+1) - t\,\text{sen}^2(t^2+1) - 2t^3\cos(t^2+1)\text{sen}(t^2+1) + t = 0$$

$$-t\cos^2(t^2+1) - t\,\text{sen}^2(t^2+1) + t = 0$$

$$-t(\cos^2(t^2+1)+\text{sen}^2(t^2+1)) + t = 0$$

$$\boxed{0=0} \to \text{recta tg en plano tg}$$

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* **Ejercicio 19**: $g: \mathbb{R} \to \mathbb{R}$ diferenciable. Param S $\quad z = y\,g(x/y)$ con $y>0$

Probar plano tg a todo pto de S pasa por origen $(0,0,0)$

$\Rightarrow$ Hallar parametrización S

una parametrización es $\vec{F}(x,y) = (x, y, y\,g(x/y))$ $\quad$ Dom $f = \{(x,y) \in \mathbb{R}^2 / y>0\}$

$\Rightarrow$ Hallar planos tg en puntos de S. $\vec{F}(x,y)$

↳ Hallar Normal en punto $\vec{F}(x_0, y_0)$

$$\vec{N}(x_0,y_0) = \frac{\partial \vec{F}}{\partial x}(x_0 y_0) \times \frac{\partial \vec{F}}{\partial y}(x_0 y_0) = \left(-\frac{\partial z}{\partial x}(x_0 y_0),\ -\frac{\partial z}{\partial y}(x_0 y_0),\ 1\right)$$

$$\frac{\partial z}{\partial x}(x_0 y_0) = y\,g'(x/y)\cdot\frac{1}{y} = g'\left(\frac{x}{y}\right)\Big|_{(x_0 y_0)} = g'(x_0/y_0)$$

$$\frac{\partial z}{\partial y}(x_0 y_0) = g(x/y) + y\cdot g'(x/y)\cdot x\left(-\frac{1}{y^2}\right)\Big|_{(x_0 y_0)} = g(x_0/y_0) - \frac{x_0}{y_0} g'(x_0/y_0)$$

$$\vec{N}(x_0,y_0) = \left(-g'(x_0/y_0),\ \frac{x_0}{y_0} g'(x_0/y_0) - g(x_0/y_0),\ 1\right)$$

↳ Hallar ec del plano $\quad \vec{N}(x_0,y_0)\,[X - \vec{F}(x_0,y_0)] = 0$

$$\vec{N}(x_0,y_0)\cdot(x-x_0,\ y-y_0,\ z-y_0\,g(x_0/y_0)) = 0$$

$$\left(-g'(x_0/y_0),\ \frac{x_0}{y_0} g'(x_0/y_0) - g(x_0/y_0),\ 1\right)(x-x_0,\ y-y_0,\ z-y_0\,g(x_0/y_0)) = 0$$

Si $(0,0,0)$ está en plano tg, entonces $\vec{N}(x_0 y_0)\,[(0,0,0)-\vec{F}(x_0 y_0)] = 0 \Rightarrow \vec{N}(x_0 y_0)\cdot -\vec{F}(x_0 y_0) = 0$

$$\left(-g'(x_0/y_0),\ \frac{x_0}{y_0} g'(x_0/y_0) - g(x_0/y_0),\ 1\right)(-x_0,\ -y_0,\ -y_0\,g(x_0/y_0)) = 0$$

$$\underbrace{x_0 g'(x_0/y_0) - y_0 \frac{x_0}{y_0} g'(x_0/y_0)}_{0} + \underbrace{y_0\,g(x_0/y_0) - y_0\,g(x_0/y_0)}_{0} = 0 \quad \to \boxed{0=0}$$

→ el plano tg en todo punto de la superficie pasa por origen

* Ejercicio 20 f clase C^∞ y $z=f(x,y)$ donde $x=2s+3t$, $y=3s-2t$ // Hallar $\frac{\partial^2 z}{\partial s^2}$, $\frac{\partial^2 z}{\partial s\partial t}$ y $\frac{\partial^2 z}{\partial t^2}$

- Ver funciones

  ↳ $f=\mathbb{R}^2\to\mathbb{R}$ puedo declarar $\vec{g}(s,t)=(2s+3t,\ 3s-2t)$

- árbol de funciones

$$
\begin{tikzcd}
 & & s \\
z=f(x,y) \arrow[r] \arrow[dr] & x \arrow[ur] \arrow[r] & t \\
 & y \arrow[r] \arrow[dr] & s \\
 & & t
\end{tikzcd}
$$

$$\frac{\partial z}{\partial s}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t}+\frac{\partial z}{\partial x}\cdot\frac{\partial y}{\partial t}$$

- Hallar las derivadas parciales primer orden, con $z=f(x,y)$

$$\frac{\partial z}{\partial s}(s,t)=\frac{\partial f}{\partial x}(x,y)\cdot\frac{\partial x}{\partial s}(s,t)+\frac{\partial f}{\partial y}(x,y)\cdot\frac{\partial y}{\partial s}(s,t)=f_x(x,y).2+f_y(x,y).3$$

$$\frac{\partial z}{\partial t}(s,t)=\frac{\partial f}{\partial x}(x,y)\cdot\frac{\partial x}{\partial t}(s,t)+\frac{\partial f}{\partial y}(x,y)\cdot\frac{\partial y}{\partial t}(s,t)=f_x(x,y).3-f_y(x,y)2$$

s depende de x y de y

- Hallar derivadas parciales segundo orden

derivar f_x s

$$\frac{\partial^2 z}{\partial^2 s}(s,t)=\frac{\partial}{\partial s}\left(\frac{\partial z}{\partial s}\right)=\frac{\partial}{\partial s}(f_x(x,y).2)+\frac{\partial}{\partial s}(f_y(x,y).3)\quad //$$

$$\frac{\partial}{\partial s}(2f_x(x,y))=2\frac{\partial}{\partial s}(f_x(x,y))=2(2f_{xx}(x,y)+3f_{xy}(x,y))=4f_{yx}(x,y)+6f_{yx}(x,y)\ ]$$

$$\frac{\partial}{\partial s}(3f_y(x,y))=3\frac{\partial}{\partial s}(f_{yx}(x,y)2+f_{yy}(x,y).3)=6f_{yx}(x,y)+9f_{yy}(x,y)$$

$$\frac{\partial^2 z}{\partial^2 s}(s,t)=4f_{xx}(x,y)+12f_{yx}(x,y)+9f_{yy}(x,y)$$

$$\frac{\partial^2 z}{\partial^2 t}(s,t)=\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial t}\right)=\frac{\partial}{\partial t}\left(3f_x(x,y)\right)-\frac{\partial}{\partial t}\left(2f_y(x,y)\right)$$

$$\frac{\partial}{\partial t}(3f_x(x,y))=3\frac{\partial}{\partial t}(f_x(x,y))=3\left(f_{xx}(x,y).3-f_{xy}(x,y)2\right)=9f_{xx}(x,y)-6f_{xy}(x,y)$$

$$\frac{\partial}{\partial t}(2f_y(x,y))=2\frac{\partial}{\partial t}(f_y(x,y))=2\left(f_{yx}(x,y)3-f_{yy}(x,y)2\right)=6f_{yx}(x,y)-4f_{yy}(x,y)$$

$$\frac{\partial^2 z}{\partial t^2}(s,t)=9f_{xx}(x,y)-6f_{xy}(x,y)-6f_{yx}(x,y)+4f_{yy}(x,y)=9f_{xx}(x,y)-12f_{xy}(x,y)+4f_{yy}(x,y)$$

$$\frac{\partial^2 z}{\partial s\partial t}=\frac{\partial}{\partial s}\left(\frac{\partial z}{\partial t}\right)=\frac{\partial}{\partial s}\left(f_x(x,y)3-f_y(x,y)2\right)$$

$$\frac{\partial}{\partial s}\left(3f_x(x,y)\right)=3\frac{\partial}{\partial s}\left(f_x(x,y)\right)=3\left(f_{xx}(x,y)2+3f_{xy}(x,y)\right)$$

$$\frac{\partial}{\partial s}\left(2f_y(x,y)\right)=2\frac{\partial}{\partial s}\left(f_y(x,y)\right)=2\left(f_{yx}(x,y)2+3f_{yy}(x,y)\right)$$

$$\frac{\partial^2 z}{\partial s\partial t}(s,t)=6f_{xx}(x,y)+\underbrace{9f_{xy}(x,y)-4f_{yx}(x,y)}_{5f_{xy}(x,y)}-6f_{yy}(x,y)$$