

* Ejercicio 34 = Verificar si es superficie, hallar expr cartesiana y graficar

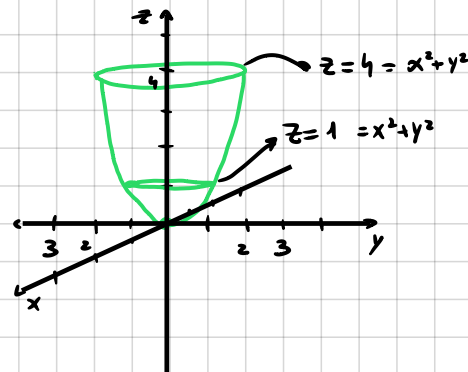
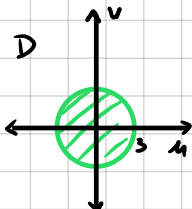
① $\vec{F}(u, v) = (u, v, u^2 + v^2)$ con $u^2 + v^2 \leq 9$

$$\left\{ \begin{array}{l} x(u, v) = u \\ y(u, v) = v \\ z(u, v) = u^2 + v^2 \end{array} \right\} \vec{F} \text{ continua en } \text{Dom } \vec{F} \rightarrow \text{es superficie } \Sigma = \text{Im } F$$

ya que sus componentes son continuas

• Desparametrizar $\rightarrow$ hallar expr cartesiana

$$\left\{ \begin{array}{l} x^2 + y^2 \leq 9 \rightarrow \text{"círculo radio 3"} \\ z = x^2 + y^2 \rightarrow \text{paraboloides} \end{array} \right.$$

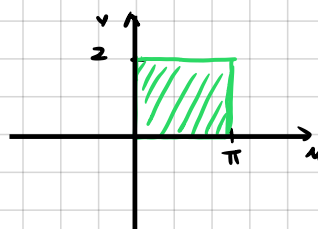


② $\vec{F}(u, v) = (2 \cos(u), 2 \sin(u), v)$ con $(u, v) \in [0, \pi] \times [0, 2]$

• Verificar superficie

$$\left\{ \begin{array}{l} x(u, v) = 2 \cos(u) \\ y(u, v) = 2 \sin(u) \\ z(u, v) = v \end{array} \right. \rightarrow F \text{ es continua ya que todas sus comp. lo son}$$

• Dominio

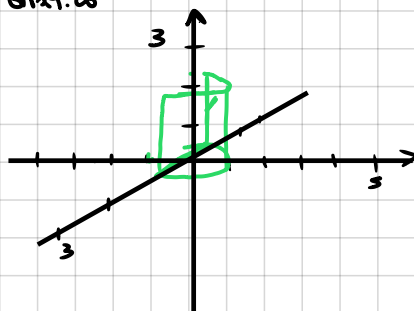


• Desparametrización

$$\left\{ \begin{array}{l} x^2 + y^2 = 2^2 \\ 0 \leq z \leq 2 \end{array} \right. \rightarrow \left\{ \begin{array}{l} x(u, v) = 2 \cos(u) \\ y(u, v) = 2 \sin(u) \\ z(u, v) = v \end{array} \right.$$

$y \geq 0 \rightarrow u \in [0, \pi]$

• Gráfico

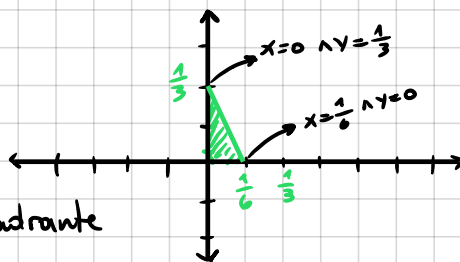


③ $\vec{F}(x, y) = (x, y, 2 - 4x - 6y)$ con $\{(x, y) \in \mathbb{R}^2 \mid 2x + 3y \leq 1, x \geq 0, y \geq 0\}$

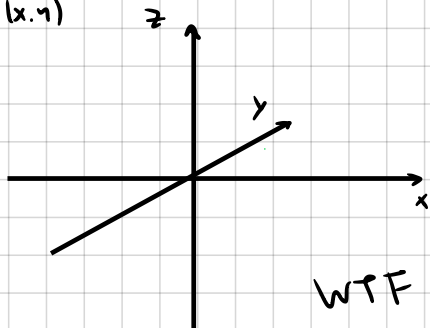
$$\left\{ \begin{array}{l} x = x \\ y = y \\ z = 2 - 4x - 6y \end{array} \right. \rightarrow \text{los componentes son continuos, así que } \vec{F} \text{ continua Im } F \text{ superficie}$$

• Gráfico dominio

$$\left\{ \begin{array}{l} 2x + 3y \leq 1 \\ y \leq \frac{1-2x}{3} \\ x \geq 0 \\ y \geq 0 \end{array} \right. \rightarrow \text{primer cuadrante}$$



• gráfico $\vec{F}(x, y)$



• expr cartesiana

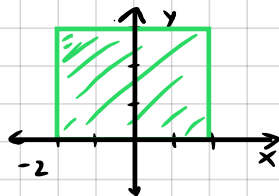
$$\left\{ \begin{array}{l} z = 2 - 4x - 6y \rightarrow \text{plano} \\ 2x + 3y \leq 1 \\ x \geq 0 \\ y \geq 0 \end{array} \right.$$

WTF

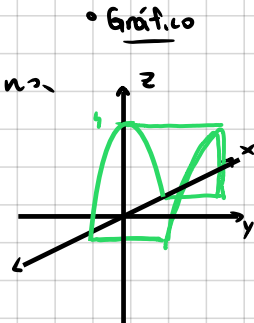
d) $\vec{F}(x, y) = (x, y, 4-x^2)$ con $(x, y) \in [-2, 2] \times [0, 3]$ • Expresión cartesiana:

$$\begin{cases} x(x, y) = x \\ y(x, y) = y \\ z(x, y) = 4-x^2 \end{cases} \rightarrow \begin{matrix} \text{todos} \\ \text{los} \\ \text{comp. son cont} \\ \text{Im } \vec{F} \text{ sup} \end{matrix}$$

• Dominio



$$\begin{cases} z = 4-x^2 \\ -2 \leq x \leq 2 \\ 0 \leq y \leq 3 \end{cases}$$



* Ejercicio 35 = Hallar ecuación vectorial para las superficies

$\mu \in [0, 2\pi]$ $v \in \mathbb{R}$

a) $x^2 + y^2 = 16$ $\rightarrow$ circunferencia en $\mathbb{R}^2$ $\rightarrow$ cilindro en $\mathbb{R}^3$

$$\begin{cases} x = 4 \cos(\mu) \\ y = 4 \sin(\mu) \\ z = v \end{cases} \quad \vec{F}(\mu, v) = (4 \cos(\mu), 4 \sin(\mu), v)$$

b) $z = xy$ $\vec{F}(\mu, v) = (\mu, v, \mu v)$, $(\mu, v) \in \mathbb{R}^2$

c) $y = x$ $\vec{F}(\mu, v) = (\mu, \mu, v)$, $(\mu, v) \in \mathbb{R}^2$

d) $x^2 - 4x + y^2 - z = 0 \rightarrow z = (x-2)^2 - 4 + y^2$ (paraboloide) $\vec{F}(\mu, v) = (\mu, v, \mu^2 - 4\mu + v^2)$

* Ejercicio 36 Determinar ecuación de líneas coord. Interpretar gráficamente

a) $\vec{F}(\mu, v) = (\cos(\mu), \sin(\mu), v)$ con $(\mu, v) \in [0, 2\pi] \times [0, 2]$

$C_{\mu=\mu_0}$ $\vec{F}(\mu_0, v) = (\cos(\mu_0), \sin(\mu_0), v)$, $\mu_0 \in [0, 2\pi]$, $v \in [0, 2] \rightarrow$ segmento de longitud 2

$C_{v=v_0}$ $\vec{F}(\mu, v_0) = (\cos(\mu), \sin(\mu), v_0)$, $v_0 \in [0, 2]$, $\mu \in [0, 2\pi] \rightarrow$ circunferencia radio 1

b) $\vec{F}(\mu, v) = (\mu, v, \mu^2 + 4v^2)$, $(\mu, v) \in \mathbb{R}^2$

$C_{\mu=\mu_0}$ $\vec{F}(\mu_0, v) = (\mu_0, v, \mu_0^2 + 4v^2)$, $\rightarrow$ parábola en $\mathbb{R}^3$ $z = \mu_0^2 + 4v^2$

$C_{v=v_0}$ $\vec{F}(\mu, v_0) = (\mu, v_0, \mu^2 + 4v_0^2)$ $\rightarrow$ parábola $z = \mu^2 + 4v_0^2$

c) $\vec{F}(x, y) = (x, y, 1+x+y)$, $(x, y) \in \mathbb{R}^2$ $\rightarrow$ plano

$C_{x=x_0}$ $\vec{F}(x_0, y) = (x_0, y, 1+x_0+y)$ $\rightarrow$ rectas $z = 1+x_0+y$

$C_{y=y_0}$ $\vec{F}(x, y_0) = (x, y_0, 1+x+y_0)$ $\rightarrow$ rectas

d) $\vec{F}(x, y) = (x, y, \sqrt{10-x^2-y^2})$, $\{(x, y) \in \mathbb{R}^2 / x^2 + y^2 \leq 9\}$

$z = \sqrt{10-x^2-y^2} \geq 0$ $\rightarrow$ cono?

$$z^2 = 10 - x^2 - y^2, z \geq 0$$

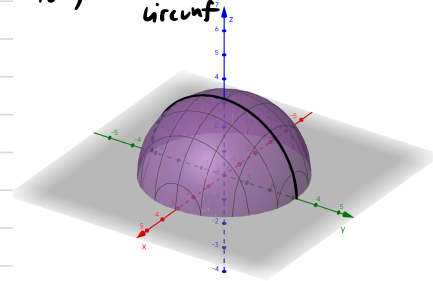
$x^2 + y^2 = 10 - z^2$ $\rightarrow$ esfera superior

circunf. que varía con z

cono $x^2 + y^2 \leq 9$

$C_{x=x_0}$ $\vec{F}(x_0, y) = (x_0, y, \sqrt{10-x_0^2-y^2})$ $\rightarrow$ arco de circunferencia

$C_{y=y_0}$ $\vec{F}(x, y_0) = (x, y_0, \sqrt{10-x^2-y_0^2})$ $\rightarrow$ arco de circunf.



* Ejercicio 37 = S $x = \vec{F}(\mu, \nu) = (\mu + \nu, \mu - \nu, \mu \nu), (\mu, \nu) \in \mathbb{R}^2$

(a) Hallar ecuación cartesiana de superficie S

$$\begin{cases} x(\mu, \nu) = \mu + \nu \\ y(\mu, \nu) = \mu - \nu \\ z(\mu, \nu) = \mu \cdot \nu \end{cases} \rightarrow \text{no importa los valores que toman, ya que al fin y al cabo, } \mu \text{ y } \nu \text{ generan todo el plano } xy$$

→ expresión cartesiana $\boxed{z = -xy}$

$$x(\mu, \nu) + y(\mu, \nu) = (\mu + \nu) + (\mu - \nu) = 2\mu$$

$$x(\mu, \nu) \cdot y(\mu, \nu) = (\mu + \nu)(\mu - \nu) = \mu^2 - \nu^2$$

$$\begin{cases} x(\mu, \nu) = \mu + \nu \\ y(\mu, \nu) = \mu - \nu \\ z(\mu, \nu) = \mu \cdot \nu \end{cases} \rightarrow \begin{cases} x = \mu + \nu \\ y = \mu - \nu \\ z = \mu \cdot \nu \end{cases} \rightarrow \begin{cases} \mu = x - \nu \\ \mu = y + \nu \\ z = \mu \cdot \nu \end{cases} \rightarrow \begin{cases} x - \nu = y + \nu \\ x = y + 2\nu \end{cases} \rightarrow \boxed{\nu = \frac{x - y}{2}}$$

$$\vec{F}_2 = \left(x, y, \frac{x^2 - y^2}{4} \right)$$

$$z = \left(\frac{x - y}{2} \right) \left(\frac{x + y}{2} \right) = \frac{x^2 - y^2}{4}$$

$$f(x, y) = \frac{x^2 - y^2}{4}$$

(b) Hallar ecuación plano tangente en $(3, -1, 2)$

→ usar parametrización $\vec{F}_2(x, y) = \left(x, y, \frac{x^2 - y^2}{4} \right) \rightarrow F(3, -1) = (3, -1, 2)$

→ Hallar Normal en $\vec{F}(x, y)$

$$\frac{\partial \vec{F}}{\partial x}(x, y) \times \frac{\partial \vec{F}}{\partial y}(x, y) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & \frac{x}{2} \\ 0 & 1 & -\frac{y}{2} \end{vmatrix} = \hat{i} \left(-\frac{x}{2} \right) - \hat{j} \left(-\frac{y}{2} \right) + \hat{k} (1) = \left(-\frac{x}{2}, \frac{y}{2}, 1 \right) \vec{N}(x, y)$$

→ Hallar normal en $\vec{F}(3, -1)$

$$\vec{N}(3, -1) = \left(-\frac{3}{2}, -\frac{1}{2}, 1 \right) \rightarrow (-3, -1, 2)$$

$$\text{plano tg} = -3x - y + 2z + 6 = 0$$

→ plano tg $\vec{N}(3, -1) [x - (3, -1, 2)] = 0 \Rightarrow (-3, -1, 2) (x - 3, y + 1, z - 2) = -3x + 9 - y + 1 + 2z - 4 = 0$

(c) Analizar recta normal int eje y

• recta normal = $\lambda \vec{N}(3, -1) + (3, -1, 2) = \lambda (-3, -1, 2) + (3, -1, 2), \lambda \in \mathbb{R}$

• recta eje y = $y | (0, 1, 0)$ intersección (6, 0, 0)

→ Hallar intersección

$$\begin{cases} -3\lambda + 3 = 0 \rightarrow \lambda = -1 \\ -\lambda - 1 = y \rightarrow \lambda = -1 \Rightarrow y = -1 \\ 2\lambda + 2 = 0 \rightarrow \lambda = -1 \end{cases} \rightarrow \text{Hay intersección en } \lambda = -1$$

(d) puntos de S con plano tangente paralelo al plano de ec $x + y + z = 0$

→ punto donde vector normal = $(1, 1, 1)$

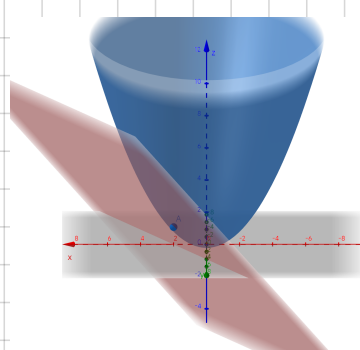
→ Hallar (x, y, z) donde $\vec{N}(x, y) = \left(-\frac{x}{2}, \frac{y}{2}, 1 \right) = (1, 1, 1)$

$$\begin{cases} -\frac{x}{2} = 1 \rightarrow x = -2 \\ \frac{y}{2} = 1 \rightarrow y = 2 \\ 1 = 1 \end{cases}$$

El punto $\vec{F}(-2, 2) = \left(-2, 2, \frac{(-2)^2 - 2^2}{4} \right) = (-2, 2, 0)$

tiene plano tg paralelo a $x + y + z = 0$

* Ejercicio 38 Hallar plano tp y recta normal



(a) Paraboloides elípticos $4x^2 + y^2 - 16z = 0$ en punto $(2, 4, 2)$

→ Parametrización $\vec{F}(x, y) = (x, y, \frac{x^2}{4} + \frac{y^2}{16}) \rightarrow \vec{F}(2, 4) = (2, 4, 1 + 1)$

→ Hallar $\vec{N}(x, y)$

$$\frac{\partial \vec{F}}{\partial x}(x, y) \times \frac{\partial \vec{F}}{\partial y}(x, y) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & \frac{x}{2} \\ 0 & 1 & \frac{y}{8} \end{vmatrix} = \left(-\frac{x}{2}, -\frac{y}{8}, 1\right) \rightarrow \vec{N}(2, 4) = (-1, -\frac{1}{2}, 1)$$

↓ multipl. $x-2$
 $(2, 1, -2)$

→ Hallar plano

$$(2, 1, -2)(x-2, y-4, z-2) = 2x + y - 2z - 4 = 0$$

→ recta Normal

$$\lambda(2, 1, -2) + (2, 4, 2), \lambda \in \mathbb{R}$$

(b) Porción de superficie cónica circular $\vec{X} = (v, 2 + \cos(\mu), 2 \sin(\mu))$, $0 \leq \mu \leq \pi$, $0 \leq v \leq 4$ $Q_0 = (2, \frac{3}{2}, \sqrt{3})$

• Hallar μ y v para $\vec{F}(\mu, v) = Q_0$

$$\begin{cases} v = 2 \\ 2 + \cos(\mu) = \frac{3}{2} \\ 2 \sin(\mu) = \sqrt{3} \end{cases} \rightarrow \begin{cases} v = 2 \\ \cos(\mu) = -\frac{1}{2} \\ \sin(\mu) = \frac{\sqrt{3}}{2} \end{cases} \rightarrow \begin{cases} v = 2 \\ \mu = \frac{2\pi}{3} \\ \mu = \frac{2\pi}{3} \end{cases}$$

• Hallar Normal $\vec{N}(\mu, v)$

$$\vec{N}(\mu, v) = \frac{\partial \vec{F}}{\partial \mu}(\mu, v) \times \frac{\partial \vec{F}}{\partial v}(\mu, v) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -\sin(\mu) & 2\cos(\mu) \\ 1 & 0 & 0 \end{vmatrix} = (0, 2\cos(\mu), \sin(\mu)) \rightarrow \vec{N}(\frac{2\pi}{3}, 2) = (0, -1, \frac{\sqrt{3}}{2})$$

• plano Normal

$$(0, -2, \sqrt{3})(x-2, y-\frac{3}{2}, z-\sqrt{3}) = -2y + \sqrt{3}z = 0$$

recta tp

$$\lambda(0, -2, \sqrt{3}) + (2, \frac{3}{2}, \sqrt{3})$$

[24/4]

(c) Porción superficie cilíndrica elíptica $\vec{X} = (v \cos(\mu), 2v, v \sin(\mu))$, $0 \leq \mu \leq \pi$, $0 \leq v \leq 3$ $Q_0 = (0, 4, 2)$

• Hallar (μ_0, v_0) para $\vec{F}(\mu_0, v_0) = Q_0$

$$\begin{cases} v_0 \cos(\mu_0) = 0 \\ 2v_0 = 4 \\ v_0 \sin(\mu_0) = 2 \end{cases} \Rightarrow \begin{cases} \cos(\mu_0) = 0 \rightarrow \mu_0 = \frac{\pi}{2} \\ v_0 = 2 \\ \sin(\mu_0) = 1 \rightarrow \mu_0 = \frac{\pi}{2} \end{cases} \quad \vec{F}(\frac{\pi}{2}, 2) = Q_0$$

• Hallar vector $\vec{N}$ en Q_0

$$\frac{\partial \vec{F}}{\partial \mu}(\mu, v) \times \frac{\partial \vec{F}}{\partial v}(\mu, v) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -v \sin(\mu) & 0 & v \cos(\mu) \\ \cos(\mu) & 2 & \sin(\mu) \end{vmatrix} = (-2v \cos(\mu), v \cos^2(\mu) + v \sin^2(\mu), -2v \sin(\mu))$$

$$\vec{N}(\mu, v) = (-2v \cos(\mu), v, -2v \sin(\mu))$$

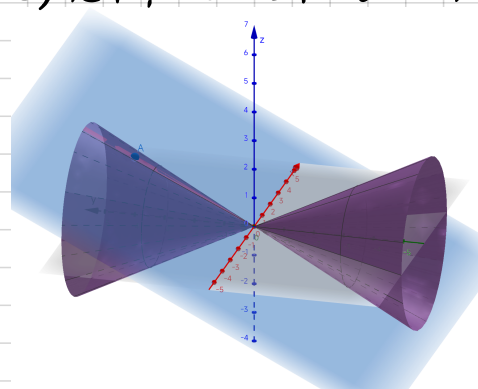
• Hallar plano tangente

$$(0, 2, -4)(x, y-4, z-2) = 0$$

• recta normal

$$\lambda(0, 2, -4) + (0, 4, 2), \lambda \in \mathbb{R}$$

$$2y - 4z = 0$$

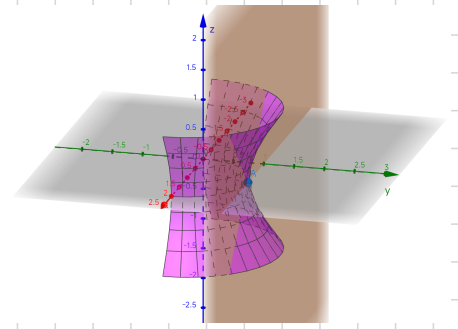


d) Porción de hipérboloide de una hoja $\vec{X} = \vec{F}(\mu, \nu) = (\cos(\mu) \cosh(\nu) + 1, \sin(\mu) \cosh(\nu), \sinh(\nu))$

$$D = [0, \pi] \times [-1, 1] \quad Q_0 = (1, 1, 0)$$

• hallar (μ, ν) para $F(\mu, \nu) = Q_0$

$$\begin{cases} \cos(\mu) \cosh(\nu) + 1 = 1 \\ \sin(\mu) \cosh(\nu) = 1 \\ \sinh(\nu) = 0 \end{cases} \Rightarrow \begin{cases} \cos(\mu) = 0 \rightarrow \mu = \frac{\pi}{2} \\ \sin(\frac{\pi}{2}) \cosh(\nu) = 1 \rightarrow \cosh(\nu) = 1 \\ \sinh(\nu) = 0 \rightarrow \nu = 0 \end{cases}$$



• Hallar Normal en Q_0

$$\frac{\partial F}{\partial \mu}(\mu, \nu) \times \frac{\partial F}{\partial \nu}(\mu, \nu) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin(\mu) \cosh(\nu) & \cos(\mu) \cosh(\nu) & 0 \\ \cos(\mu) \sinh(\nu) & \sin(\mu) \sinh(\nu) & \cosh(\nu) \end{vmatrix} = \begin{pmatrix} \cos^2(\mu) \cosh(\nu) & -\sin^2(\mu) \cosh(\nu) & -\sin^2(\mu) \cosh(\nu) \sinh(\nu) - \cos^2(\mu) \cosh(\nu) \sinh(\nu) \end{pmatrix}$$

la normal en $Q_0 = \vec{0}$
 no es punto regular,
 no admite plano tg
 ni recta normal

$$\vec{N}(\mu, \nu) = (\cos^2(\mu) \cosh(\nu), -\sin^2(\mu) \cosh(\nu) \sinh(\nu), -\cosh(\nu) \sinh(\nu))$$

$$\vec{N}(\frac{\pi}{2}, 0) = (0, 0, 0)$$

• Hallar derivadas parciales

$$\frac{\partial F}{\partial \mu}(\mu, \nu) = (-\sin(\mu) \cosh(\nu), \cos(\mu) \cosh(\nu), 0) \Rightarrow \frac{\partial F}{\partial \mu}(\frac{\pi}{2}, 0) = (-1, 0, 0)$$

$$\frac{\partial F}{\partial \nu}(\mu, \nu) = (\cos(\mu) \sinh(\nu), \sin(\mu) \sinh(\nu), \cosh(\nu)) \Rightarrow \frac{\partial F}{\partial \nu}(\frac{\pi}{2}, 0) = (0, 0, 1)$$

• Hallar Normal en Q_0

$$\vec{N}_0 = \frac{\partial F}{\partial \mu}(\frac{\pi}{2}, 0) \times \frac{\partial F}{\partial \nu}(\frac{\pi}{2}, 0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (0, 1, 0)$$

• Hallar plano tg

$$(0, 1, 0)(x-1, y-1, z) = 0$$

$$\boxed{y-1=0}$$

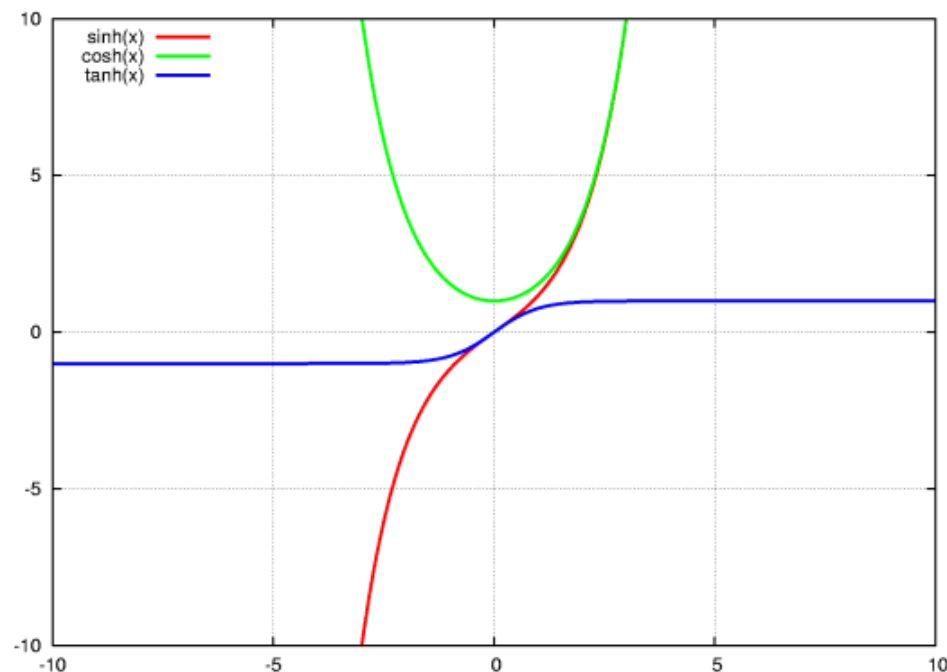
• Hallar recta normal

$$r(\lambda) = \lambda(0, 1, 0) + (1, 1, 0), \lambda \in \mathbb{R}$$

De este gráfico sabemos que =

$$- \cosh(x) > 0 \quad \forall x$$

$$- \sinh(x) = 0, \quad x = 0$$



* Ejercicio 39 = $\varphi: D \rightarrow \mathbb{R}^3 / \varphi(\mu, \nu) = (\nu, 2\cos(\mu), 2\sin(\mu))$, $D = [0, 2\pi] \times [0, 1]$

$\downarrow$
 $\mu \in [0, 2\pi], \nu \in [0, 1]$

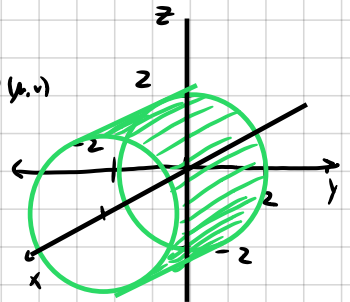
① Demostrar φ permite parametrizar porción de sup cilíndrica

$$\varphi(\mu, \nu) = \begin{cases} x(\mu, \nu) = \nu \\ y(\mu, \nu) = 2\cos(\mu) \\ z(\mu, \nu) = 2\sin(\mu) \end{cases} \quad \begin{array}{l} \rightarrow \text{todas las componentes} \\ \text{de } \varphi(\mu, \nu) \text{ son} \\ \text{continuas en } D \end{array} \quad \begin{array}{l} \rightarrow \varphi \text{ es continuo en } D \\ \downarrow \\ \text{la Im}(\varphi) \text{ es una sup.} \end{array}$$

• Desparametrizar para graficar

$$\begin{cases} x = \nu \\ y = 2\cos(\mu) \\ z = 2\sin(\mu) \end{cases}, \quad 0 \leq x \leq 1 \quad \Rightarrow \quad \begin{cases} 0 \leq x \leq 1 \\ y^2 + z^2 = 4 \end{cases}$$

• Gráfico $\varphi(\mu, \nu)$



② Hallar ecuación plano tangente a sup en punto $P = (1, \sqrt{3}, 1)$ y ver intersección con ejes de coord

• Hallar (μ_0, ν_0) donde $\varphi(\mu_0, \nu_0) = P$

$$\begin{cases} \nu_0 = 1 \\ 2\cos(\mu_0) = \sqrt{3} \\ 2\sin(\mu_0) = 1 \end{cases} \Rightarrow \begin{cases} \nu_0 = 1 \\ \cos(\mu_0) = \frac{\sqrt{3}}{2} \\ \sin(\mu_0) = \frac{1}{2} \end{cases} \Rightarrow \mu_0 = \frac{\pi}{6} \quad \rightarrow \varphi\left(\frac{\pi}{6}, 1\right) = (1, \sqrt{3}, 1)$$

• Hallar derivadas parciales

$$\frac{\partial \varphi}{\partial \mu}(\mu, \nu) = (0, -2\sin(\mu), 2\cos(\mu)) \rightarrow \frac{\partial \varphi}{\partial \mu}\left(\frac{\pi}{6}, 1\right) = (0, -1, \sqrt{3})$$

$$\frac{\partial \varphi}{\partial \nu}(\mu, \nu) = (1, 0, 0) \rightarrow \frac{\partial \varphi}{\partial \nu}\left(\frac{\pi}{6}, 1\right) = (1, 0, 0)$$

• Hallar Normal $\vec{N}$ en P

$$\vec{N} = \frac{\partial \varphi}{\partial \mu}\left(\frac{\pi}{6}, 1\right) \times \frac{\partial \varphi}{\partial \nu}\left(\frac{\pi}{6}, 1\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & \sqrt{3} \\ 1 & 0 & 0 \end{vmatrix} = (0, \sqrt{3}, 1)$$

• Hallar el plano tg

$$(0, \sqrt{3}, 1) \cdot (x-1, y-\sqrt{3}, z-1) = 0$$

$$\boxed{\sqrt{3}y + z - 4 = 0}$$

• Probar si plano $\sqrt{3}y + z - 4 = 0$ tiene int con recta $(x, 0, 0) \rightarrow$ eje x

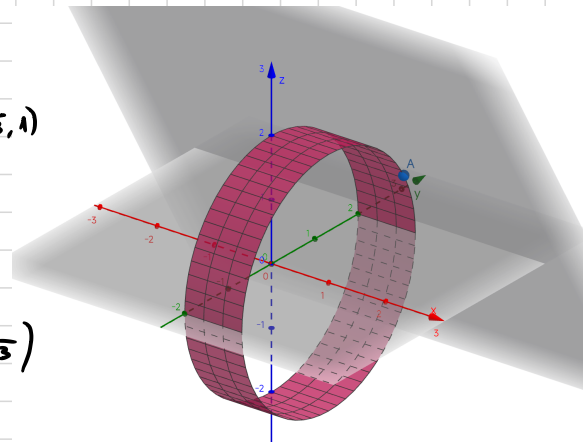
$$\rightarrow \sqrt{3} \cdot 0 + 0 - 4 = 0 \rightarrow \boxed{-4 = 0} \rightarrow \text{No tiene intersección con eje x}$$

• Probar intersección $\sqrt{3}y + z - 4 = 0$ con eje y $\rightarrow (0, y, 0) = y(0, 1, 0)$

$$\sqrt{3} \cdot y + 0 - 4 = 0 \rightarrow \boxed{y = \frac{4}{\sqrt{3}}} \rightarrow \text{plano intersecciona eje y en punto } (0, \frac{4}{\sqrt{3}}, 0)$$

• Probar intersección $\sqrt{3}y + z - 4 = 0$ con eje z $\rightarrow (0, 0, z)$

$$\sqrt{3} \cdot 0 + z - 4 = 0 \rightarrow \boxed{z = 4} \rightarrow \text{plano intersecciona eje z en punto } (0, 0, 4)$$



* Ejercicio 40: $\vec{x} = \vec{r}(\mu) = (1+2\mu, 3\mu-1, 5\mu)$, $\mu \in \mathbb{R}$ rect normal a sup Σ en $A=(5,5,10)$

↓ Analizar si plano tp a Σ en A tiene pta intersección con eje x

• Hallar vector normal

$$\vec{r}(\mu) = (1+2\mu, 3\mu-1, 5\mu) = (1, -1, 0) + \mu(2, 3, 5) \quad \vec{N}_0 = (2, 3, 5)$$

• Hallar ec plano tp a Σ en A

$$\vec{N}(x-A)=0 \Rightarrow (2,3,5)(x-5, y-5, z-10)=0$$

$$2x+3y+5z-75=0$$

• Chequear intersección plano tp con recta $(x,0,0)$

$$2x+0+0-75=0 \rightarrow x = \frac{75}{2} = 37,5 \quad \text{Intersección con eje x en punto } \left(\frac{75}{2}, 0, 0\right)$$