14/4

DIFERENCIABILIDAD

* Ejercicio 18 : Hallar matriz jacobiana de los campos, y los gradientes

(a) $\vec{F}(x,y) = (3x^2y,\ x-y)$ → Diferenciable ya que f_1 y f_2 diferenciable

- $f_1(x,y) = 3x^2y$

$\frac{\partial f_1}{\partial x}(x,y) = 6xy \qquad \frac{\partial f_1}{\partial y}(x,y) = 3x^2 \qquad \nabla f_1(x,y) = (6xy,\ 3x^2)$

- $f_2(x,y) = x-y$

$$J_f = \begin{pmatrix} 6xy & 3x^2 \\ 1 & -1 \end{pmatrix}$$

$\frac{\partial f_2}{\partial x}(x,y) = 1 \qquad \frac{\partial f_2}{\partial y}(x,y) = -1 \qquad \nabla f_2(x,y) = (1,-1)$

(b) $G(x) = (x^2+1,\ 2x)$, $g_1 = x^2+1$, $g_2 = 2x$

$\frac{\partial g_1}{\partial x}(x) = 2x \qquad \frac{\partial g_2}{\partial x}(x) = 2 \qquad JG = \begin{bmatrix} 2x \\ 2 \end{bmatrix}$

(c) $h(x,y,z) = xy + z^2x$

$\frac{\partial h}{\partial x}(x,y,z) = y+z^2 \qquad \frac{\partial h}{\partial y}(x,y,z) = x \qquad \frac{\partial h}{\partial z}(x,y,z) = 2zx \qquad J_h(x,y,z) = \nabla h(x,y,z) = (y+z^2,\ x,\ 2zx)$

(d) $\vec{L}(x,y) = (x^2y,\ y,\ x-xy)$ $\qquad L_1(x,y) = x^2y \qquad L_2(x,y) = y \qquad L_3(x,y) = x-xy$

- $L_1(x,y) = x^2y$

$\frac{\partial L_1}{\partial x}(x,y) = 2xy \qquad \frac{\partial L_1}{\partial y}(x,y) = x^2 \qquad \nabla L_1(x,y) = (2xy,\ x^2)$

- $L_2(x,y) = y$

$\frac{\partial L_2}{\partial x}(x,y) = 0 \qquad \frac{\partial L_2}{\partial y}(x,y) = 1 \qquad \nabla L_2(x,y) = (0,1)$

$$J_L(x,y) = \begin{pmatrix} 2xy & x^2 \\ 0 & 1 \\ 1-y & x \end{pmatrix}$$

- $L_3(x,y) = x-xy$

$\frac{\partial L_3}{\partial x}(x,y) = 1-y \qquad \frac{\partial L_3}{\partial y}(x,y) = -x \qquad \nabla L_3(x,y) = (1-y,\ x)$

(f) $f(x,y) = x\,\text{sen}(2x-y)$

$\frac{\partial f}{\partial x}(x,y) = \text{sen}(2x-y) + 2\cos(2x-y)\,x$

$\nabla f(x,y) = (\text{sen}(2x-y) + 2\cos(2x-y)x,\ -x\cos(2x-y))$

$\frac{\partial f}{\partial y}(x,y) = x\cos(2x-y)\,(-1)$

ⓔ $\vec{N}(x,y,z) = (2xy,\ x^2 - ze^y)$ $\quad N_1(x,y,z) = 2xy \quad N_2 = x^2 - ze^y$

- $N_1(x,y,z) = 2xy$

$\frac{\partial N_1}{\partial x}(x,y,z) = 2y \qquad \frac{\partial N_1}{\partial y}(x,y,z) = 2x \qquad \frac{\partial N_1}{\partial z}(x,y,z) = 0 \qquad \nabla N_1(x,y,z) = (2y,\ 2x,\ 0)$

- $N_2(x,y,z) = x^2 - ze^y$

$\frac{\partial N_2}{\partial x}(x,y,z) = 2x \qquad \frac{\partial N_2}{\partial y}(x,y,z) = -ze^y \qquad \frac{\partial N_2}{\partial z}(x,y,z) = -e^y \qquad \nabla N_2(x,y,z) = (2x,\ -ze^y,\ -e^y)$

$$J_N(x,y,z) = \begin{pmatrix} 2y & 2x & 0 \\ 2x & -ze^y & -e^y \end{pmatrix}$$

* Ejercicio 19 $\quad f(x,y) = x\sqrt{xy}$ y $A = (1,4) \in D^\circ$

ⓐ Hallar razones suficientes para asegurar diferenciabilidad de f en A → ¿Calcular x definición?

$f(1,4) = 1 \cdot \sqrt{4} = 2$

$\frac{\partial f}{\partial x}(x,y) = \sqrt{xy} + \frac{1}{2\sqrt{xy}} \cdot xy \longrightarrow \frac{\partial f}{\partial x}(1,4) = \sqrt{4} + \frac{4}{2\sqrt{4}} = 2 + 1 = 3$

$\frac{\partial f}{\partial y}(x,y) = \frac{x}{2\sqrt{xy}} \cdot x = \frac{x^2}{2\sqrt{xy}} \longrightarrow \frac{\partial f}{\partial y}(1,4) = \frac{1^2}{2\sqrt{4}} = \frac{1}{4}$

si es diferenciable: $\lim_{(x,y)\to(1,4)} \frac{f(x,y) - \left[f(1,4) + \frac{\partial f}{\partial x}(1,4)(x-1) + \frac{\partial f}{\partial y}(1,4)(y-4) \right]}{\sqrt{(x-1)^2 + (y-4)^2}} = 0$

$\boxed{\text{IND}}$

$$\lim_{(x,y)\to(1,4)} \frac{x\sqrt{xy} - \left[2 + 3(x-1) + \frac{1}{4}(y-4)\right]}{\sqrt{(x-1)^2 + (y-4)^2}} = \lim_{(x,y)\to(1,4)} \frac{x\sqrt{xy} - 2 - 3x + 3 - \frac{1}{4}y + 1}{\sqrt{(x-1)^2 + (x-4)^2}} = \lim_{(x,y)\to(1,4)} \frac{\overbrace{x\sqrt{xy} - 3x - \frac{y}{4} + 2}^{\to 0}}{\underbrace{\sqrt{(x-1)^2 + (x-4)^2}}_{\to 0}}$$

$$f(x,y) = \underbrace{f(A) + \frac{\partial f}{\partial x}(A)(x - x_0) + \frac{\partial f}{\partial y}(A)(y - y_0)}_{L(x)} + \underbrace{\mu(H)\,\|H\|}_{\lim_{H\to 0}\ \to 0}$$

$f(x,y) = 2 + 3(x-1) + \frac{1}{4}(y-4)$

$\underbrace{f(1,4)}_{2} = 2 + 3(1-1) + \frac{1}{4}(4-4) \longrightarrow \boxed{2 = 2}$ → la aproximación lineal es igual a $f(A) = L(A)$

ⓑ → Hallar derivadas direccionales máximas, mínimas y nulas $\quad \check{r} = (a,b),\ a^2 + b^2 = 1$

$\nabla f(1,4) = (3, \frac{1}{4})$

$\frac{\partial f}{\partial \check{r}_{max}}(1,4) = \nabla f(1,4) \cdot \check{r} = \|\nabla f(1,4)\| \underbrace{\|\check{r}\| \cos(\theta)}_{1}$, $\theta = 0$ $\longrightarrow r_{max} = \frac{\nabla f(1,4)}{\|\nabla f(1,4)\|} = \left(\frac{12}{\sqrt{145}}, \frac{1}{\sqrt{145}}\right)$ $\quad \boxed{\frac{\partial f}{r_{max}}(1,4) = 17}$

$\frac{\partial f}{\partial r_{min}}(1,4) = \nabla f(1,4) \cdot \check{r} = \|\nabla f(1,4)\| \underbrace{\|\check{r}\| \cos(\theta)}_{-1} = -17 \quad r_{min} = -\left(\frac{12}{\sqrt{145}}, \frac{1}{\sqrt{145}}\right)$

$\frac{\partial f}{\partial r_0}(1,4) = \nabla f(1,4) \cdot \check{r} = \begin{matrix} 3a + \frac{1}{4}b = 0 \\ b = -12a \end{matrix} \quad \Big\| \quad \begin{matrix} a^2 + (-12a)^2 = 1 \\ 145a^2 = 1 \to a = \pm\frac{1}{\sqrt{145}} \end{matrix} \quad \check{r}_0 = \left(\frac{1}{\sqrt{145}}, \frac{-12}{\sqrt{145}}\right) \quad \check{r}_2 = \left(\frac{-1}{\sqrt{145}}, \frac{12}{\sqrt{145}}\right)$

*** Ejercicio 20** $\quad f \in C^1$, $f'(A, (0.6, 0.8)) = 2$ y $f'(A, (0.8, 0.6)) = 5$

→ Hallar máxima derivada direccional de f en A y en qué dirección

$\check{r}_1 = (0,6 , 0,8)$. $\check{r}_2 = (0,8 , 0,6)$ $\qquad \underbrace{f'(A, \check{r}_1) = 2 \quad f'(A, \check{r}_2) = 5}$ transformación lineal

$\nabla f(A) = (a, b)$

- Hallar $\nabla f(A)$ sabiendo que

$$f'(A, \check{r}_1) = \nabla f(A) \cdot \check{r}_1 = \quad 0,6a + 0,8b = 2$$
$$f'(A, \check{r}_2) = \nabla f(A) \cdot \check{r}_2 = \quad 0,8a + 0,6b = 5$$

$$\Rightarrow \left(\begin{array}{cc|c} 0,6 & 0,8 & 2 \\ 0,8 & 0,6 & 5 \end{array}\right) \xrightarrow{F_2 \to F_2 - \frac{4}{3}F_1} \left(\begin{array}{cc|c} 0,6 & 0,8 & 2 \\ 0 & -\frac{7}{15} & \frac{7}{3} \end{array}\right)$$

$$\begin{cases} 0,6\,a + 0,8\,b = 2 \longrightarrow a = \dfrac{2 - 0,8(-5)}{0,6} = 10 \\ -\frac{7}{15} b = \frac{7}{3} \longrightarrow \boxed{b = -5} \end{cases}$$

$\nabla f(A) = (10, -5)$

- $$\frac{\partial f}{\partial r_{max}}(A) = \|\nabla f(A)\| = \sqrt{(-5)^2 + (10)^2} = \sqrt{125} = 5\sqrt{5}$$

$$r_{max} = \frac{\nabla f(A)}{\|\nabla f(A)\|} = \frac{(10, -5)}{\sqrt{125}} = \left(\frac{10}{\sqrt{125}}, \frac{-5}{\sqrt{125}}\right)$$
$$\left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$$

15/4

*** Ejercicio 21** $\quad f: D \subset \mathbb{R}^2 \to \mathbb{R}$ diferenciable en $P_0 = (x_0, y_0) \subset D^\circ$

↳ mostrar $\vec{N_0} = \left(-\frac{\partial f}{\partial x}(P_0), -\frac{\partial f}{\partial y}(P_0), 1\right)$ es normal a gráfica f en $Q_0 = (x_0, y_0, f(x_0, y_0))$

$\Rightarrow \vec{N_0}$ normal a Q_0 ⟶ $\vec{N_0}$ es múltiple de la Normal del plano tangencial en Q_0

- Hallar plano tangencial a Q_0 ⟶ plano dif en P_0

$$L(x, y) = f(P_0) + \frac{\partial f}{\partial x}(P_0)(x - x_0) + \frac{\partial f}{\partial y}(P_0)(y - y_0)$$ ⟶ plano tangencial a P.

$$z = f(P_0) + \frac{\partial f}{\partial x}(P_0)(x - x_0) + \frac{\partial f}{\partial y}(P_0)(y - y_0)$$

$$\underbrace{\frac{\partial f}{\partial x}(P_0)(x - x_0)}_{Ax} + \underbrace{\frac{\partial f}{\partial y}(P_0)(y - y_0)}_{By} \underbrace{- z}_{Cz} + \underbrace{f(P_0)}_{D} = 0$$

vector normal al plano tiene la forma $\left(\frac{\partial f}{\partial x}(P_0), \frac{\partial f}{\partial x}(P_0), -1\right)$

*** Cómo probar diferenciabilidad**

- $f(x + H) = f(A) + \frac{\partial f}{\partial x}(A)h + \frac{\partial f}{\partial y}(A)k + \|H\|\mu(H)$ donde $\lim\limits_{H \to 0} \mu(H) \overbrace{\|H\|}^{\in \mathbb{R}} = 0$? por def

- Probar que es C^1 ⟶ entonces es diferenciable

- Probar que No es diferenciable ⟶ no es continua
  ⟶ no es derivable ⟶ $\nexists$ derivadas parciales
  ⟶ $\nexists$ derivadas parciales continuas

* Ejercicio 22 : - Justificar diferenciabilidad f en P_0

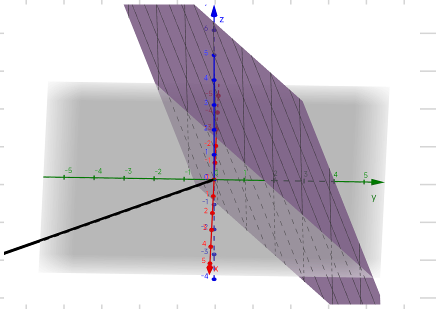
- Representar gráfico de f, plano tg a sup Q_0 y vector $\vec{N_0}$ aplicado Q_0

ⓐ $f(x,y) = 5 + 2x - 3y$, $P_0 = (0,0)$

- Probar diferenciabilidad ⟶ probar clase C1

$D_f = \mathbb{R}^2$ → continua en todo su dom

$\frac{\partial f}{\partial x}(x,y) = 2$ $\quad \frac{\partial f}{\partial y}(x,y) = -3$ $\Rightarrow$ función f es de clase C^1 por lo tanto es diferenciable en su dominio → $P_0 \in D_f$



- Hallar gráfico, plano tg, vector tg en $P_0 = (0,0)$

$\Rightarrow$ plano $z = \underbrace{f(0,0)}_{} + \frac{\partial f}{\partial x}(0,0)\,x + \frac{\partial f}{\partial y}(0,0)\,y$

$z = 5 + 2x - 3y$

$\Rightarrow$ Normal $\vec{N_0} = (2,-3,-1)$ ⟶ recta normal $\lambda(2,3,-1) + (0,0,5)$

ⓑ $f(x,y) = x^2 - 2x + y^2$, $P_0 = (-1,2)$

- Hallar Dominio $D_f = \mathbb{R}^2$

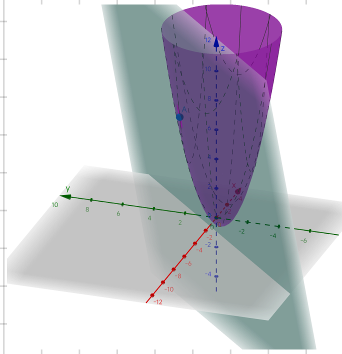
- Verificar clase de función

$\frac{\partial f}{\partial x}(x,y) = 2x - 2$ $\quad \frac{\partial f}{\partial y}(x,y) = 2y$ $\Rightarrow$ $D_{f_x} \subset D_f$, $D_{f_y} \subset D_f$

$\frac{\partial f}{\partial x}(-1,2) = -2-2 = -4$ $\quad \frac{\partial f}{\partial y}(-1,2) = 4$

f es de C^1 por lo que es diferenciable en todo su dom

como $P_0 \in D_f$ entonces P_0 es diferenciable



- Hallar plano tangente

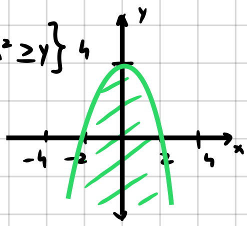
$L(x,y) = \underbrace{f(-1,2)}_{} + \frac{\partial f}{\partial x}(-1,2)(x+1) + \frac{\partial f}{\partial y}(-1,2)(y-2)$

$z = 7 + (-4)(x+1) + 4(y-2)$ $\Rightarrow \vec{N_0} = (-4, 4, -1)$

ⓒ $f(x,y) = \sqrt{4 - x^2 - y^2}$, $P_0 = (1,-1)$

- Ver dominio $D_f = \{(x,y) \in \mathbb{R}^2 / 4 - x^2 \geq y\}$

$4 - x^2 - y \geq 0 \rightarrow 4 - x^2 \geq y$



- Probar diferenciabilidad

$\frac{\partial f}{\partial x}(x,y) = \frac{1}{2\sqrt{4-x^2-y^2}} \cdot 2x$ $\quad \frac{\partial f}{\partial y}(x,y) = \frac{1}{2\sqrt{4-x^2-y^2}} - 2y$

$D_{f_x} \subset D_f$ $\quad D_{f_y} \subset D$

Como f es de clase C^1 es diferenciable en su dominio $D_f = \{(x,y) \in \mathbb{R}^2 / 4 - x^2 \geq y\}$

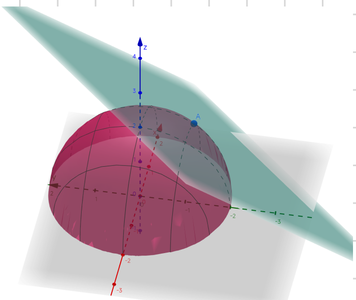
como $P_0 = (1,-1) \in D_f$ entonces P_0 diferenciable

$f(1,-1) = \sqrt{4-1-1} = \sqrt{2}$ $\quad \frac{\partial f}{\partial x}(1,-1) = \frac{-1}{\sqrt{2}}$ $\quad \frac{\partial f}{\partial y}(1,-1) = \frac{1}{\sqrt{2}}$

- Ecuación plano tg

$L(x,y) = f(1,-1) + \frac{\partial f}{\partial x}(1,-1)(x-1) + \frac{\partial f}{\partial y}(1,-1)(y+1)$

$z = \sqrt{2} - \frac{\sqrt{2}}{2}(x-1) + \frac{\sqrt{2}}{2}(y+1)$



* Ejercicio 23

: Sabiendo gráfico $f(x,y)$ tiene plano tg $\pi_1 = 3x+y+2z=6$ en $A=(2,1,z_0)$

↳ Ver si la recta normal al gráfico $f(x,y)$ en A intersecta sup $y=x^2$

⇒ Gráfico de f $G_f = (x, y, f(x,y))$

⇒ Plano tg a G_f en A $= G_{tg} = (x, y, L(x,y))$

⇒ Si plano es tg a G_f entonces coinciden en A : $G_f = G_{tg} = A = (2,1,z_0)$

$$(x,y,f(x,y)) = (x,y,L(x,y)) = (2,1,z_0)$$

$$\underbrace{(2,1,f(2,1))}_{G_f} = \underbrace{(2,1,L(2,1))}_{G_{Tg}} = (2,1,z_0)$$

- Hallar $z = L(x,y)$

$$\pi = 3x+y+2z=6 \longrightarrow z = \frac{6-3x-y}{2} = 3-\frac{3}{2}x-\frac{1}{2}y$$

- Hallar z_0

$$z_0 = f(2,1) = L(2,1) = 3 - 3\cdot\frac{2}{2} - \frac{1}{2}\cdot 1 = -\frac{1}{2}$$

- Armar recta normal $\vec{r}_n(t)$

↳ Hallar normal al plano = $\vec{N} = (3,1,2)$

↳ Hallar punto A → $A = (2,1,-\frac{1}{2})$

$$\vec{r}_n(t) = t(3,1,2) + (2,1,-\tfrac{1}{2})$$

$$\vec{r}_n(t)\begin{cases} x = 3t+2 \\ y = t+1 \\ z = 2t-\frac{1}{2}\end{cases}$$

- Verificar $\vec{r}_n(t) \cap y=x^2$

$$(t+1) = (3t+2)^2$$

$$(t+1) = 3^2t^2 + 12t + 4$$

$$0 = 9t^2 + 11t + 3$$

$$t_1, t_2 = \frac{-11 \pm \sqrt{11^2-4(9)(3)}}{18}$$

$$t_1 = \frac{-11+\sqrt{13}}{18} \qquad t_2 = \frac{-11-\sqrt{13}}{18}$$

Hay dos intersecciones de la recta normal al sup $y=x^2$

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* Ejercicio 24

$L: \mathbb{R}^n \to \mathbb{R}$ $\quad L(x) = \sum_{i=1}^{n} c_i x_i$ (TL), demostrar L diferenciable $\forall A \in \mathbb{R}^n$, $\nabla L(A) = (c_1 \dots c_n)$

↳ y deriv direcc en A en $\check{r}$ = $\underline{L'(A,\check{r}) = L(\check{r})}$

- Demostrar diferenciabilidad → Demostrar clase C^1

↳ Si L es función lineal entonces es clase C^∞, diferenciable en su dom $\forall A \in D$ y $D \subset \mathbb{R}^n$

- Demostrar $\nabla \vec{L}(A) = (c_1, c_2 \dots c_n)$

↓ armar ∇L

$$\frac{\partial L_1}{\partial x_1}(x) = c_1 \quad \frac{\partial L_2}{\partial x_2} = c_2 \quad \dots \quad \frac{\partial L_n}{\partial x_n} = c_n \quad , \text{ donde } \{c_1 \dots c_n\} \in \mathbb{R}$$

$$\nabla L(A) = \left(\frac{\partial L}{\partial x_1}; \frac{\partial L}{\partial x_2} \dots \frac{\partial L}{\partial x_n}\right)$$

$$\nabla L(A) = (c_1, c_2 \dots c_n) \quad \forall A \in D \subset \mathbb{R}^n$$

- Demostrar $L'(A,\check{r}) = L(\check{r})$, $\check{r} = (a_1 \dots a_n) \in \mathbb{R}^n$

$$L'(A,\check{r}) = \nabla L(A)\cdot \check{r} = (c_1, c_2, \dots, c_n)(a_1, a_2, \dots, a_n) = \underbrace{c_1 a_1 + c_2 a_2 + \dots + c_n a_n}_{\sum_{i=1}^{n} c_i a_i \,=\, L(\check{r})}$$

por lo que $L'(A,\check{r}) = L(\check{r})$

* Ejercicio 25: $f: \mathbb{R}^2 \to \mathbb{R}$ diferenciable en todo $\mathbb{R}^2$

↳ plano tangente a sup en $A = \left(1, -\frac{1}{2}, f\left(1, -\frac{1}{2}\right)\right) \Rightarrow 2x - 4y - 2z = 6$

↳ Hallar $f\left(1, -\frac{1}{2}\right)$ y $\nabla f\left(1, -\frac{1}{2}\right)$

- Hallar $f\left(1, -\frac{1}{2}\right)$

Si el plano es tangente a la gráfica de la función, entonces, tienen el mismo punto $f(x,y) = L(x,y)$ en $(x,y) = \left(1, -\frac{1}{2}\right)$

↳ Hallar $L(x,y)$

$$2x - 4y - 2z = 6 \rightarrow z = \frac{2x - 4y - 6}{2} = x - 2y - 3 \rightarrow z = L(x,y) = x - 2y - 3$$

→ Hallar $f\left(1, -\frac{1}{2}\right) = L\left(1, -\frac{1}{2}\right) = 1 - 2\left(-\frac{1}{2}\right) - 3 = 1 + 1 - 3 = -1 \Rightarrow f\left(1, -\frac{1}{2}\right) = -1$

→ Hallar $\nabla f\left(1, -\frac{1}{2}\right) = \left(\frac{\partial f}{\partial x}\left(1, -\frac{1}{2}\right), \frac{\partial f}{\partial y}\left(1, -\frac{1}{2}\right)\right) = (1, -2)$

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* Ejercicio 26 $f(x,y) = xye^{x+y}$ Hallar puntos con plano tg paralelo a plano xy

- Ecuación del plano xy → $N = k(0,0,1)$ → $0x + 0y + kz + D = 0$

↳ hallar algún punto donde su plano tangente tengan Normal $= k(0,0,1)$ $\frac{\partial f}{\partial x}(x,y) = 0 \wedge \frac{\partial f}{\partial y}(x,y) = 0$

- Hallar derivadas parciales

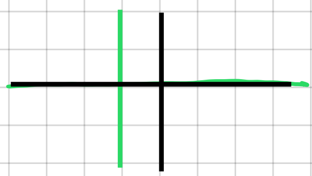
$$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}(xy)\, e^{x+y} + \frac{\partial}{\partial x}(e^{x+y}) \cdot xy = ye^{x+y} + xye^{x+y} = ye^{x+y}(1+x)$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial y}(xy)\, e^{x+y} + \frac{\partial}{\partial y}(e^{x+y}) \cdot xy = xe^{x+y} + e^{x+y}xy = xe^{x+y}(1+y)$$

- Hallar puntos (x,y) donde $\frac{\partial f}{\partial x}(x,y) = 0$ $\{(x,y) \in \mathbb{R}^2 / x = -1\} \cup \{(x,y) \in \mathbb{R}^2 / y = 0\}$

$ye^{x+y}(1+x) = 0 \Rightarrow ye^{x+y} = 0 \vee (1+x) = 0$

$\boxed{y = 0} \vee \underbrace{e^{x+y}}_{\neq 0\ \forall (x,y) \in \mathbb{R}^2} = 0$ $\boxed{x = -1}$

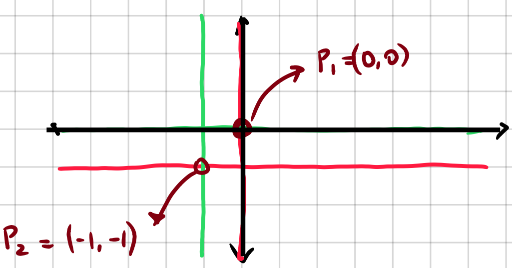


- Hallar puntos (x,y) donde $\frac{\partial f}{\partial y}(x,y) = 0$ $\{(x,y) \in \mathbb{R}^2 / y = -1\} \cup \{(x,y) \in \mathbb{R}^2 / x = 0\}$

$xe^{x+y}(1+y) = 0 \Rightarrow xe^{x+y} = 0 \vee (1+y) = 0$

$\boxed{x = 0} \vee \underbrace{e^{x+y} = 0}_{\phi}$ $\boxed{y = -1}$

- Hallar puntos que satisfacen $\frac{\partial f}{\partial x}(x,y) = 0$ y $\frac{\partial f}{\partial y}(x,y) = 0$ al mismo tiempo



en P_1 y P_2 el plano tg es paralelo a plano xy

↓

puntos gráfico

$(0, 0, f(0,0))$ y $(-1, -1, f(-1,-1)) = (-1, -1, e^{-2})$

21/4

* **Ejercicio 27**: $f(x,y,z) = e^{xz+xy^2}$. Hallar valor aproximado $f(0.1, 0.98, 2.05)$ con aproximación lineal

→ Hallar aproximación lineal de $f(0,1,2)$ con $L(0,1,2) = f(0,1,2) + \frac{\partial f}{\partial x}(0,1,2)x + \frac{\partial f}{\partial y}(y-1) + \frac{\partial f}{\partial z}(z-2)$

↳ Hallar derivadas parciales

$$\frac{\partial f}{\partial x}(x,y,z) = (z+y^2)\,e^{xz+xy^2} \longrightarrow \frac{\partial f}{\partial x}(0,1,2) = (2+1^2)e^0 = 3$$

$$\frac{\partial f}{\partial y}(x,y,z) = 2xy\,e^{xz+xy^2} \longrightarrow \frac{\partial f}{\partial y}(0,1,2) = (2.0.1)e^0 = 0$$

$$\frac{\partial f}{\partial z}(x,y,z) = x\,e^{xz+xy^2} \longrightarrow \frac{\partial f}{\partial y}(0,1,2) = 0\,e^0 = 0$$

↳ Hallar $f(0,1,2) = e^{0.2+0.1^2} = e^0 = 1$

$\Rightarrow L(x,y,z) = 1 + 3x + 0(y-1) + 0(z-2)$

$L(x,y,z) = 1 + 3x$

↳ Calcular $f(0.1, 0.98, 2.05) \cong L(0.1, 0.98, 2.05) = 1 + 3.(0.1) = 1,3 \Rightarrow f(0.1, 0.98, 2.05) \cong 1,3$

* **Ejercicio 28**: $f: \mathbb{R}^2 \to \mathbb{R}$ diferenciable / $f(1,2) = 5$ $\quad \underbrace{\text{versor } \check{r}_{max} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}_{\text{en } (1,2)}$ $\quad f'_x(1,2) = 3\sqrt{2}$

(a) Hallar ecuación plano tg a la gráf f en $(1,2,f(1,2)) \longrightarrow$ Hallar $L(1,2)$ / tg $(1,2,L(1,2)) = (1,2,f(1,2))$

$L(1,2) = f(1,2) + \frac{\partial f}{\partial x}(1,2)(x-1) + \frac{\partial f}{\partial y}(1,2)(y-2) \longrightarrow$ falta hallar $\frac{\partial f}{\partial y}(1,2) = f'_y(1,2)$

• Sabemos que $\check{r}_{max} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$\Rightarrow \check{r}_{max} = \frac{\nabla f(1,2)}{\|\nabla f(1,2)\|} \qquad \nabla f(1,2) = \Big(\underbrace{\frac{\partial f}{\partial x}(1,2)}_{3\sqrt{2}}, \underbrace{\frac{\partial f}{\partial y}(1,2)}_{\mu}\Big) \longrightarrow \|\nabla f(1,2)\| = \sqrt{\underbrace{\left[\frac{\partial f}{\partial x}(1,2)\right]^2}_{(3\sqrt{2})^2} + \underbrace{\left[\frac{\partial f}{\partial y}(1,2)\right]^2}_{(\mu)^2}}$$

$\mu = \frac{\partial f}{\partial y}(1,2)$

$$\Rightarrow \check{r}_{max} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{(3\sqrt{2})^2 + \mu^2}} \cdot (3\sqrt{2}, \mu) = \frac{1}{\sqrt{18+\mu^2}}(3\sqrt{2}, \mu)$$

sistemas de ecuaciones

$$\Rightarrow \begin{cases} \frac{3\sqrt{2}}{\sqrt{18+\mu^2}} = \frac{1}{\sqrt{2}} \\ \frac{\mu}{\sqrt{18+\mu^2}} = \frac{1}{\sqrt{2}} \end{cases} \rightarrow \begin{cases} \left(\frac{3\sqrt{2}}{\sqrt{18+\mu^2}}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 \\ \left(\frac{\mu}{\sqrt{18+\mu^2}}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 \end{cases} \Rightarrow \begin{cases} \frac{18}{18+\mu^2} = \frac{1}{2} \\ \frac{\mu^2}{18+\mu^2} = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} 36 - 18 = \mu^2 \longrightarrow \mu = \sqrt{18} = 3\sqrt{2} \\ 2\mu^2 - \mu^2 = 18 \rightarrow \mu = \sqrt{18} = 3\sqrt{2} \end{cases}$$

$\mu = \frac{\partial f}{\partial y}(1,2) = 3\sqrt{2}$

• $L(1,2) = \underset{f(1,2)}{5} + \underset{\frac{\partial f}{\partial x}(1,2)}{3\sqrt{2}}(x-1) + \underset{\frac{\partial f}{\partial y}(1,2)}{3\sqrt{2}}(y-2) \Rightarrow$ plano tangente $(1,2,L(1,2)) = (1,2,f(1,2))$

(b) Hallar valor aprox $f(1.01, 1.98)$

$f(1.01, 1.98) \cong L(1.01, 1.98) = 5 + 3\sqrt{2}(0,1) + 3\sqrt{2}(-0,02) \cong 5,34$

$f(1,01, 1.98) \cong 5,34$

* Ejercicio 29: Demostrar $\overbrace{\ln\left(2x+\frac{1}{y}\right)}^{f(x,y)} \cong \overbrace{1+2x-y}^{g(x,y)}$ en entorno $(x_0, y_0) = (0,1)$

- Hallar aproximación lineal y comparar
- Hallar $L_1(x,y)$ aproximación lineal $f(x,y)$ en $(0,1)$

↳ $f(0,1) = \ln(2.0 + 1/1) = \ln(1) = 0$

↳ $\frac{\partial f}{\partial x}(x,y) = \frac{1}{2x+1/y}\cdot 2 \longrightarrow \frac{\partial f}{\partial x}(0,1) = \frac{1}{1}\cdot 2 = 2$

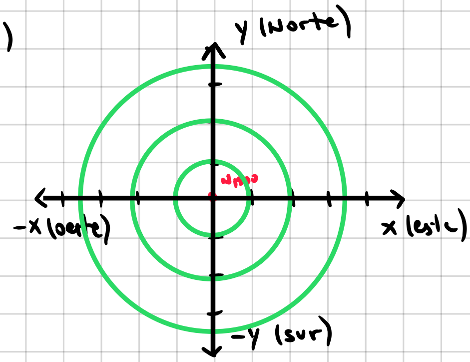
↳ $\frac{\partial f}{\partial y}(x,y) = \frac{1}{2x+1/y}\cdot -\frac{1}{y^2} \longrightarrow \frac{\partial f}{\partial y}(0,1) = \frac{1}{0+1}\cdot\frac{-1}{1^2} = -1$

$g(x,y)$ es una aproximación lineal de $f(x,y)$ en entorno $(x_0, y_0) = (0,1)$

$$L(x,y) = \underbrace{f(0,1)}_{0} + \frac{\partial f}{\partial x}(0,1)(x-0) + \frac{\partial f}{\partial y}(0,1)(y-1) = 0 + 2(x-0) + (-1)(y-1) = \underbrace{2x - y + 1}_{g(x,y)}$$

* Ejercicio 31: $f(x,y) = 1500\, e^{-(x^2+y^2)/200}$ x por (este) y por (Norte)

(a) Hallar y dibujar algunas curvas de nivel (conj nivel) de f



$\mathrm{Dom}\, f = \{(x,y) \in \mathbb{R}^2\}$

→ Conjunto nivel 1500 $N_{1500} = \{(0,0)\}$

$1500\, e^{-(x^2+y^2)/200} = 1500$

$\frac{-x^2-y^2}{200} = 0$

$\underbrace{x^2}_{\geq 0\ \forall x \in \mathbb{R}} = \underbrace{-y^2}_{\leq 0\ \forall y \in \mathbb{R}} \to (0,0)$

↳ Conjunto nivel 1 $N_1 =$

$1500\, e^{-(x^2+y^2)/200} = 1$

$-(x^2+y^2) = \ln\left(\frac{1}{1500}\right)\cdot 200$

$x^2+y^2 = -\ln\left(\frac{1}{1500}\right)\cdot 200 \cong 1462,64$

$x^2+y^2 \cong 38,24^2$

- Conjunto Nivel 500 $N_{500} = \{(x,y)\in\mathbb{R}^2 \,/\, x^2+y^2 \cong 219,72\}$

$1500\, e^{-(x^2+y^2)/200} = 500$

$x^2+y^2 = -\ln\left(\frac{1}{3}\right)\cdot 200 \cong 219,72$

$x^2+y^2 \cong 14,82^2$

- Conj Nivel 1000 $N_{1000} = \{(x,y)\in D \,/\, x^2+y^2 \cong 81,09\}$

$1500\, e^{-(x^2+y^2)/200} = 1000$

$x^2+y^2 = -\ln\left(\frac{2}{3}\right)\cdot 200 \cong 81,09$

$x^2+y^2 \cong 9^2$

(b) alpinista en pto $\left(10, 10, \overbrace{1500/e}^{f(10,10)}\right)$. Si se mueve hacia noreste ¿asciende o desciende? Hallar pendiente

↳ Si va hacia noreste (NE), entonces avanza hacia dirección $y=x$ → un vector en esa dirección y sentido es $\vec{r} = (1,1)$

↳ Hallar versor de $(1,1)$ Hallar derivada direccional y ver si pendiente es positivo o negativo

$\check{r} = \frac{1}{\sqrt{2}}(1,1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

- Hallar $\nabla f(10,10) = \left(\frac{\partial f}{\partial x}(10,10), \frac{\partial f}{\partial y}(10,10)\right)$

$\frac{\partial f}{\partial x}(x,y) = 1500\, e^{-(x^2+y^2)/200}\cdot\left(\frac{-1}{200}\right)2x \to \frac{\partial f}{\partial x}(10,10) = -\frac{150}{e}$

$\frac{\partial f}{\partial y}(x,y) = 1500\, e^{-(x^2+y^2)/200}\cdot\left(\frac{-1}{200}\right)2y \to \frac{\partial f}{\partial x}(10,10) = -\frac{150}{e}$

$\nabla f(10,10) = \left(-\frac{150}{e}, -\frac{150}{e}\right)$

- Hallar derivada direccional

$\frac{\partial f}{\partial \check{r}}(10,10) = \nabla f(10,10)\cdot\check{r}$

$= \left(-\frac{150}{e}, -\frac{150}{e}\right)\cdot\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\frac{\partial f}{\partial \check{r}}(10,10) = 2\cdot\left(\frac{-75}{e}\right) = \frac{-150}{e}$

RTA: Si avanza NE estando $(10, 10, 1500/e)$ desciende con pendiente $\frac{-150}{e}$ negativa

∗ Ejercicio 32 rectángulo $a = 10 \pm 0{,}1$ $b = 100 \pm \Delta b$

de manera proporcional — misma tasa de error

mínimo error? — una buena aproximación

↳ Hallar Δb para que contribución al $\underline{\Delta A = \Delta a\, b + \Delta b\, a}$ sea mismo orden

→ Incerteza medición indirecta rectángulo

• Fig análisis

$A = a.b = 1000$

$\Delta A = \Delta a b + \Delta b\, a$ ↗ no sabemos valor de ΔA

pero una aproximación lineal es $\Delta A' \cong \frac{\partial A}{\partial a}.\underbrace{(a)}_{\Delta a} + \frac{\partial A}{\partial b}\underbrace{(b)}_{\Delta b}$

$\frac{\partial A}{\partial a} = b \quad \frac{\partial A}{\partial b} = a$

$\boxed{\Delta A' \cong b\Delta a + a\Delta b}$

Ejercicio a chequear

Qué sea mismo orden

↳ misma tasa de error

$|b\Delta a = a\Delta b| \rightarrow 100 . 0{,}1 = 10 . \Delta b \rightarrow \boxed{\Delta b = 1}$

∗ Ejercicio 33 = $T = 2\pi\sqrt{l/g}$, l longitud hilo, $\vec{g}$ gravedad

hallar error abs y rel → porcentaje error

↳ Hallar cotas para errores para hallar $\vec{g}$ con $T = 2s \pm 0{,}02s$, $l = 1m \pm 0{,}001m$

$$\Rightarrow \begin{cases} T = 2\pi\sqrt{l/g} \\ \Delta T = 2\pi\sqrt{\Delta l/\Delta g} \end{cases} \Rightarrow \begin{cases} \frac{T}{2\pi} = \sqrt{l/g} \\ \frac{\Delta T}{2\pi} = \sqrt{\Delta l/g} \end{cases} \Rightarrow \begin{cases} \frac{T^2}{(2\pi)^2} = |l/g| \\ \frac{\Delta T^2}{(2\pi)^2} = \left|\frac{\Delta l}{\Delta g}\right| \end{cases} \begin{cases} |g| = \frac{(2\pi)^2 l}{T^2} \\ |\Delta g| = \frac{\Delta l . (2\pi)^2}{\Delta T^2} \end{cases} \Rightarrow ?$$

$$\boxed{|\vec{g}| = \frac{(2\pi)^2 l}{T^2}}$$

gravedad en función de l y T → $g(1m, 2s) = \frac{(2\pi)^2 . (1m)}{(2s)^2} = \pi^2 \frac{m}{s} \cong 9{,}86\ m/s$

$\vec{g} + \Delta\vec{g} =$ → Aproximación lineal

error

$$\Delta g = \frac{\partial g}{\partial l}(l,T)\Delta l + \frac{\partial g}{\partial T}(l,T)\Delta T \rightarrow \underbrace{\Delta g = \frac{2\pi^2}{T^2}\Delta l - \frac{(2\pi)^2 l\, 2}{T^3}\Delta T}$$

$$\frac{\partial g}{\partial l}(l,T) = \frac{(2\pi)^2}{T^2} \qquad \frac{\partial g}{\partial T}(l,T) = \frac{-(2\pi)^2 l\, 2T}{T^4} = \frac{-(2\pi)^2 l\, 2}{T^3}$$

$(\pi = 3{,}1416)$

$$\Delta g(1m, 2s) = \frac{(2\pi)^2}{(2s)^2} 0{,}001m - \frac{(2\pi)^2 (1m) . 2}{(2s)^3} . 0{,}02s$$

$$\Delta g(1m, 2s) = \frac{(2\pi)^2}{(2s)^2}(0{,}001m - 0{,}02s) \cong -0{,}18$$

↓ Corrección

$$\Delta g = \underbrace{\left|\frac{\partial g}{\partial l}\right|\Delta l + \left|\frac{\partial g}{\partial T}\right|\Delta T}_{\text{van en módulo}}$$

porque si hay Δl o ΔT neg, no quiero que reste, sino que sume

error relativo

$$\frac{\Delta g}{g} . 100 = \frac{0{,}207\ m/s}{9{,}86\ m/s} . 100 = \boxed{2{,}09\%}$$

$$\Delta g(1m, 0{,}02s) = \frac{(2\pi)^2}{(2s)^2}(0{,}001m + 0{,}02s) \cong 0{,}207$$