* Ejercicio 10 = Hallar, si es posible, k / f sea continua en $\mathbb{R}^2$

$$f(x,y) = \begin{cases} \dfrac{x^3 - x(y+1)^2}{x^2+(y+1)^2} & \text{si } (x,y) \neq (0,-1) \\ k & \text{si } (x,y) = (0,-1) \end{cases}$$

→ $f(0,-1) = k$ → existe imagen → Hallar $k \in \mathbb{R}$ / $\lim\limits_{(x,y)\to(0,-1)} f(x,y) = f(0,-1) = k$

→ $\lim\limits_{(x,y)\to(0,-1)} \dfrac{\overset{\to 0}{x^3 - x(y+1)^2}}{\underset{\to 0}{x^2+(y+1)^2}} = \text{Ind } \frac{0}{0}$

- Intentar transformar expresión

$$\frac{x^3-x(y+1)^2}{x^2+(y+1)^2} = \frac{x[x^2-(y+1)^2]}{x^2+(y+1)^2} = x\underbrace{\left(\frac{x^2-(y+1)^2}{x^2+(y+1)^2}\right)}$$

División polinómica

$$\begin{array}{l|l} x^2 - (y+1)^2 & x^2+(y+1)^2 \\ \hline -x^2+(y+1)^2 & 1 \\ 0 - 2(y+1)^2 & \end{array}$$

$$x\left(-\frac{2(y+1)^2}{x^2+(y+1)^2} + 1\right)$$

$$x\left[\frac{(x-(y+1))(x+(y+1))}{x^2+(y+1)^2}\right]$$ ⇒ No sirve simplificar expr

$$\lim_{(x,y)\to(0,-1)} \frac{x^3-x(y+1)^2}{x^2+(y+1)^2} = \lim_{(x,y)\to(0,-1)} x\left(\frac{x^2-(y+1)^2}{x^2+(y+1)^2}\right) = \lim_{(x,y)\to(0,-1)} x\left(\frac{x^2}{x^2+(y+1)^2} - \frac{(y+1)^2}{x^2+(y+1)^2}\right)$$

$$= \lim_{(x,y)\to(0,-1)} \overset{\to 0}{x}\,.\underbrace{\frac{x^2}{x^2+(y+1)^2}}_{\substack{\text{acotado}\\ B^*((0,-1),\varepsilon)}} - \lim_{(x,y)\to(0,-1)} \overset{\to 0}{x}\underbrace{\frac{(y+1)^2}{x^2+(y+1)^2}}_{\text{acotado}} = 0 - 0 = 0 \to \boxed{k=0}$$

DERIVADA FUNCIONES VARIAS VARIABLES

* Ejercicio 10 = Hallar derivadas parciales y evaluar en punto

(a) $f(x,y) = xy + x^2$, en $(2,0)$

~~rect tg x~~ ~~⇒ $f(2,0) + \overset{4}{\sim}(x-2)$~~

- $\dfrac{\partial f}{\partial x}(x,y) = y + 2x$, $\dfrac{\partial f}{\partial x}(2,0) = 4$
- $\dfrac{\partial f}{\partial y}(x,y) = x$, $\dfrac{\partial f}{\partial y}(2,0) = 2$

(b) $f(x,y) = \operatorname{senh}(x^2+y)$, en $(1,-1)$

- $\dfrac{\partial f}{\partial x}(x,y) = \cosh(x^2+y)\,.\,2x$, $\dfrac{\partial f}{\partial x}(1,-1) = \cosh(0)\,.\,2 = 2$
- $\dfrac{\partial f}{\partial y}(x,y) = \cosh(x^2+y)$, $\dfrac{\partial f}{\partial y}(1,-1) = \cosh(0) = 1$

ⓒ $f(x,y,z) = \dfrac{xz}{y+z}$, en $(1,1,1)$

- $\dfrac{\partial f}{\partial x}(x,y,z) = \dfrac{z}{y+z}$, $\dfrac{\partial f}{\partial x}(1,1,1) = \dfrac{1}{2}$
- $\dfrac{\partial f}{\partial y}(x,y,z) = xz \cdot \dfrac{-1}{(y+z)^2}$, $\dfrac{\partial f}{\partial y}(1,1,1) = \dfrac{-1}{4}$
- $\dfrac{\partial f}{\partial z}(x,y,z) = \dfrac{\partial}{\partial z}\left(\dfrac{xz}{y+z}\right) = \dfrac{(x)(y+z)-(1)(xz)}{(y+z)^2} = \dfrac{xy+xz-xz}{(y+z)^2} = \dfrac{xy}{(y+z)^2}$. $\dfrac{\partial f}{\partial z}(1,1,1) = \dfrac{1}{4}$

ⓓ $f(x,y,z) = \ln(1+x+y^2z)$, en $(1,2,0)$ y $(0,0,0)$

$\dfrac{\partial f}{\partial x}(x,y,z) = \dfrac{\partial}{\partial x}\left(\ln(1+x+y^2z)\right) = \dfrac{1}{1+x+y^2z}$, $\dfrac{\partial f}{\partial x}(1,2,0) = \dfrac{1}{2}$, $\dfrac{\partial f}{\partial x}(0,0,0) = 1$

$\dfrac{\partial f}{\partial y}(x,y,z) = \dfrac{1}{1+x+y^2z} \cdot \dfrac{\partial}{\partial y}\left(1+x+y^2z\right) = \dfrac{2zy}{1+x+y^2z}$, $\dfrac{\partial f}{\partial y}(1,2,0) = 0$ $\dfrac{\partial f}{\partial y}(0,0,0) = 0$

$\dfrac{\partial f}{\partial z}(x,y,z) = \dfrac{y^2}{1+x+y^2z}$, $\dfrac{\partial f}{\partial z}(1,2,0) = 2$ $\dfrac{\partial f}{\partial z}(0,0,0) = 0$

ⓔ $f(x,z) = \operatorname{sen}(x\sqrt{z})$, $\left(\frac{\pi}{3}, 4\right)$

$\dfrac{\partial f}{\partial x}(x,z) = \cos(x\sqrt{z})\sqrt{z}$. $\dfrac{\partial f}{\partial x}\left(\frac{\pi}{3},4\right) = \left(-\frac{1}{2}\right)\sqrt{4} = -1$

$\dfrac{\partial f}{\partial z}(x,z) = \cos(x\sqrt{z})\dfrac{x}{2\sqrt{z}}$ $\dfrac{\partial f}{\partial z}\left(\frac{\pi}{3},4\right) = \dfrac{\pi}{3}\cdot\left(\frac{-1}{2}\right)\cdot\dfrac{1}{4} = -\dfrac{\pi}{24}$

ⓕ $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$, en $(-3,4)$

- $\dfrac{\partial f}{\partial x}(x,y) = \left(u^{\frac{-1}{2}}\right)' = \dfrac{-1}{2}u^{\frac{-3}{2}}(u') = \dfrac{-1}{2}\cdot\dfrac{1}{\sqrt{(x^2+y^2)^3}}\cdot 2x$, $\dfrac{\partial f}{\partial x}(-3,4) = \dfrac{-1}{2}\dfrac{1}{\sqrt{(9+16)^3}}\cdot -6 = \dfrac{3}{125}$

  $u = x^2+y^2$

- $\dfrac{\partial f}{\partial y}(x,y) = \dfrac{-1}{2}\cdot\dfrac{1}{\sqrt{(x^2+y^2)^3}}\cdot 2y$, $\dfrac{\partial f}{\partial y}(-3,4) = \dfrac{-4}{\sqrt{(25)^3}} = \dfrac{-4}{125}$

ⓖ $f(x,y) = \displaystyle\int_x^y \operatorname{sen}(\ln(1+t^3))\, dt$, en $(1,2)$

$\displaystyle\int \operatorname{sen}(\ln(1+t^3))dt$ $\Rightarrow$ Atrás:

⑧ $f(x,y) = \int_x^{y^2} \operatorname{sen}(\ln(1+t^3))\, dt$, en $(1,2)$

$\exists G(t) \,/\, G'(t) = \operatorname{sen}(\ln(1+t^3))$ $\qquad f(x,y) = G(y^2) - G(x)$

$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}\left(\underbrace{G(y^2)}_{\in \mathbb{R}} - G(x)\right) = -G'(x)\underbrace{\frac{\partial}{\partial x}(x)}_{1} = -\operatorname{sen}(\ln(1+x^3))$ $\frac{\partial f}{\partial x}(1,2) = -\operatorname{sen}(\ln(1+1)) = -\operatorname{sen}(\ln(2))$

$\frac{\partial f}{\partial y}(x,y) = \frac{\partial}{\partial y}\left(G(y^2) - \underbrace{G(x)}_{\in\mathbb{R}}\right) = G'(y^2)\frac{\partial}{\partial y}(y^2) = \operatorname{sen}(\ln(1+y^6))\, 2y$ $\frac{\partial f}{\partial x}(1,2) = \operatorname{sen}(\ln(65))\, 4$

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* Ejercicio 12 = Analizar derivada parcial en $(0,0)$ y analizar continuidad

ⓐ $f(x,y) = \begin{cases} \frac{2x^3-y^3}{x^2+3y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

$\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}(x,y) = 2$

• Analizar derivadas parciales en $(0,0)$

derivada continua

$\lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0}\left[\frac{2h^3-0}{h^2+0} - 0\right]\cdot\frac{1}{h} = \lim_{h\to 0}\frac{2h^3}{h^3} = 2 = \frac{\partial f}{\partial x}(0,0)$

$\lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0}\left[\frac{0-h^3}{0+3h^2} - 0\right]\cdot\frac{1}{h} = \lim_{h\to 0}\frac{-h^3}{3h^3} = -\frac{1}{3} = \frac{\partial f}{\partial y}(0,0)$

• Analizar continuidad f en $(0,0)$

$f(0,0) = 0 \longrightarrow$ existe imagen

$\lim_{(x,y)\to(0,0)} \frac{2x^3-y^3}{x^2+3y^2} = \text{Ind } \frac{0}{0} \Rightarrow \lim_{(x,y)\to(0,0)} \frac{2x^3-y^3}{x^2+3y^2} = \lim_{(x,y)\to(0,0)} \frac{x^3+x^3-y^3}{x^2+3y^2}$?

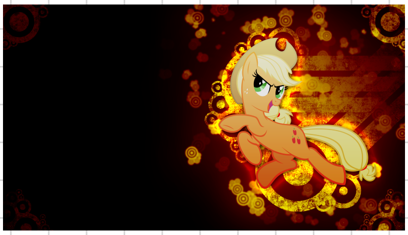
$$\begin{array}{l|l} 2x^3 - y^3 & x^2+3y^2 \\ \hline -\;2x^3+6y^2x & 2x - \frac{1}{3}y + 2yx \\ \quad 0 - y^3 - 6y^2x & \\ -\;-y^3 - \frac{1}{3}yx^2 & \\ \quad 0 - 6y^2x + \frac{1}{3}yx^2 & \\ -\;+6y^2x + 2yx^3 & \\ \quad 0 \quad 2yx^3 + \frac{1}{3}yx^2 & \end{array}$$

• probar recta $y = mx$

$\lim_{x,y\to(0,0)} \frac{x^3(2-m^3)}{\underbrace{x^2}_{\to 0}\underbrace{(1+3m^2)}_{\in\mathbb{R}}} = 0$

KHE Creo que se puede reemplazar por coord polar

$\boxed{\begin{array}{l} x = r\cos(\alpha) \\ y = r\operatorname{sen}(\alpha) \end{array}}$ $\boxed{?}$? ? ? ? ?



這一個不好玩了ㄨ

想了好久

(b) $f(x,y) = \begin{cases} \dfrac{x^2-2y^2}{x-y} & x \neq y \\ 0 & \text{cualquier otro caso} \end{cases}$

• Hallar derivadas parciales en $(0,0)$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \left[\frac{h^2-0}{h-0} - 0\right] \cdot \frac{1}{h} = 1$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0} \left[\frac{0-2h^2}{0-h} - 0\right] \cdot \frac{1}{h} = 2$$

• Hallar continuidad en $(0,0)$ de f

• $f(0,0) = 0 \to$ existe imagen

• $\lim\limits_{(x,y)\to(0,0)} \dfrac{\overset{\to 0}{x^2-2y^2}}{\underset{\to 0}{x-y}} \underset{\text{ind } \frac{0}{0}}{=} \lim\limits_{(x,y)\to(0,0)} \left[\dfrac{x^2-y^2}{x-y} - \dfrac{y^2}{x-y}\right] = \lim\limits_{(x,y)\to(0,0)} \dfrac{\cancel{(x-y)}\overset{\to 0}{(x+y)}}{\cancel{(x-y)}} - \lim\limits_{(x,y)\to(0,0)} \dfrac{y^2}{x-y}$

• probar rectas $y=mx$

$$\lim_{x\to 0} \frac{x^2-2m^2x^2}{x-mx} = \lim_{x\to 0} \frac{\overset{\to 0}{x-2m^2x}}{\underset{\to 1-m}{1-m}} = 0$$

Probar $y = (x-x^2) \to$ porque denominador es $y-x$

$$\lim_{x\to 0} \frac{x^2-2(x-x^2)^2}{x-(x-x^2)} = \lim_{x\to 0} \frac{x^2-2(x^2-2xx^2+x^4)}{x^2}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}-2\cancel{x^2}(1-2x+x^2)}{\cancel{x^2}} = \lim_{x\to 0} \overset{\to -1}{1-2}\overset{\to 1}{(1-2x+x^2)} = -1$$

f no cont

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(c) $f(x,y) = \begin{cases} 0 & \text{cuando } xy \neq 0 \\ 1 & \text{cuando } xy = 0 \end{cases}$

• derivadas parciales en $(0,0)$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{1-1}{h} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{f(0,0+h)-f(0,0)}{h} = \lim_{h\to 0} \frac{1-1}{h} = 0$$

• Hallar continuidad f en $(0,0)$

- $f(0,0) = 1$
- $\lim\limits_{(x,y)\to(0,0)} f(x) = 0$

} f no continua en $(0,0)$

* Ejercicio 13 = Det dom not, obtener derivadas seg. orden. Indicar Dom derivada.

ⓐ $f(x,y) = \ln(x^2+y)$ $Dom = \{(x,y) \in \mathbb{R}^2 \,/\, x^2+y > 0\}$

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{x^2+y} \cdot 2x$$

$D = \{(x,y) \in \mathbb{R} \,/\, x^2+y > 0\}$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{2(x^2+y) - 4x^2}{(x^2+y)^2}$$

$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{0(1) - 2x}{(x^2+y)^2} = \frac{-2x}{(x^2+y)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{1}{x^2+y}$$

$$\frac{\partial^2 f}{\partial y^2}(x,y) = \frac{-1}{(x^2+y)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{-2x}{(x^2+y)^2} = \frac{\partial^2 f}{\partial y \partial x}(x,y)$$

teorema de Schwartz
$\exists \frac{\partial f}{\partial x}(x,y)$ continua
$\exists \frac{\partial f}{\partial y \partial x}(x,y)$ continua

ⓑ $f(x,y,z) = x\,\text{sen}(y) + y\cos(z)$ $Dom\ \{(x,y,z) \in \mathbb{R}^3\}$

$$\frac{\partial f(x,y,z)}{\partial x} = \text{sen}(y)$$

$$\frac{\partial^2 f}{\partial x^2}(x,y,z) = 0$$

$$\frac{\partial^2 f}{\partial y \partial x}(x,y,z) = \cos(y) = \frac{\partial^2 f}{\partial x \partial y}(x,y,z)$$

$$\frac{\partial^2 f}{\partial z \partial x}(x,y,z) = 0 = \frac{\partial^2 f}{\partial x \partial z}(x,y,z)$$

Teorema Schwartz son $f \in C^\infty$

$$\frac{\partial f}{\partial y}(x,y,z) = x\cos(y) + \cos(z)$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y,z) = \cos(y) = \frac{\partial^2 f}{\partial y \partial x}(x,y,z)$$

$$\frac{\partial^2 f}{\partial y^2}(x,y,z) = -x\,\text{sen}(y)$$

$$\frac{\partial^2 f}{\partial z \partial y}(x,y,z) = -\text{sen}(z)$$

$$\frac{\partial f}{\partial z}(x,y,z) = -y\,\text{sen}(z)$$

$$\frac{\partial^2 f}{\partial x \partial z}(x,y,z) = \frac{\partial^2 f}{\partial z \partial x}(x,y,z) = 0$$

$$\frac{\partial^2 f}{\partial y \partial z}(x,y,z) = \frac{\partial^2 f}{\partial z \partial y}(x,y,z) = -\text{sen}(z)$$

$$\frac{\partial^2 f}{\partial z^2}(x,y,z) = -y\cos(z)$$

(c) $f(x,y) = \operatorname{arctg}\left(\frac{x}{y}\right)$ $\quad$ Dom $\{(x,y)\in\mathbb{R}^2 / y \neq 0\}$ $\quad$ recordar $\frac{d}{dx}(\operatorname{arctg}(x)) = \frac{1}{1+x^2}$

$$\frac{\partial f}{\partial x}(x,y) = \underbrace{\frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\left(\frac{1}{y}\right)}_{\frac{1}{y+\frac{x^2}{y}}} \longrightarrow \frac{\partial^2 f}{\partial x^2}(x,y) = \left(\frac{1}{y}\right)\cdot\frac{-2\left(\frac{x}{y}\right)}{\left(1+\left(\frac{x}{y}\right)^2\right)^2} = \frac{-\frac{2x}{y}}{\left(y+\frac{x^2}{y}\right)^2}$$

$$\frac{\partial^2 f}{\partial y\partial x}(x,y) = \frac{-\frac{\partial}{\partial y}\left(y+\frac{x^2}{y}\right)}{\left[y+\frac{x^2}{y}\right]^2} = \frac{-\left[1+x^2\frac{\partial}{\partial y}\left(\frac{1}{y}\right)\right]}{\left(y+\frac{x^2}{y}\right)^2} = \frac{\frac{x^2}{y^2}-1}{\left(y+\frac{x^2}{y^2}\right)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \underbrace{\frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\left(\frac{-x}{y^2}\right)}_{\frac{-x}{y^2+x^2}} \longrightarrow \frac{\partial^2 f}{\partial x\partial y} = \frac{\frac{\partial}{\partial x}(x)(y^2+x^2) - \frac{\partial}{\partial x}(y^2+x^2)(x)}{(y^2+x^2)^2} = \frac{(y^2+x^2)-2x^2}{(y^2+x^2)^2}$$

(?)

WATAFAK SKIBIDI

(?) rehacer !!

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(d) $f(x,y) = \sqrt{\frac{x^2}{a^2}+\frac{y^2}{b^2}}$ $\quad$ Dom $=\{(x,y)\in\mathbb{R}^2 / \frac{x^2}{a^2}+\frac{y^2}{b} \geq 0\}$

$$\frac{-1}{4\sqrt{\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^3}}\left(\frac{2x}{a^2}\right)^2 + \frac{1}{a^2\sqrt{\frac{x^2}{a^2}+\frac{y^2}{b^2}}}$$

$$\frac{\partial f}{\partial x}(x,y) = \underbrace{\frac{1}{2\sqrt{\frac{x^2}{a^2}+\frac{y}{b^2}}}\cdot\left(\frac{2x}{a^2}\right)}_{\frac{1}{2}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{1}{2}}\left(\frac{2x}{a^2}\right)} \longrightarrow \frac{\partial^2 f}{\partial x^2}(x,y) = \frac{1}{2}\left[\frac{-1}{2}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{3}{2}}\left(\frac{2x}{a^2}\right)\left(\frac{2x}{a^2}\right) + \frac{2}{a^2}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{1}{2}}\right]$$

$$\frac{\partial^2 f}{\partial y\partial x}(x,y) = \frac{x}{a^2}\frac{\partial}{\partial y}\left[\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{1}{2}}\right] = \frac{x}{a^2}\left(-\frac{1}{2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{3}{2}}\frac{2y}{b^2}$$

$$= \frac{-xy}{a\,b^2}\cdot\frac{1}{\sqrt{\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^3}}$$

T. Schwartz

$$\frac{\partial f}{\partial y}(x,y) = \underbrace{\frac{1}{2\sqrt{\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)}}\cdot\left(\frac{2y}{b^2}\right)}_{\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{1}{2}}\left(\frac{y}{b^2}\right)} \longrightarrow \frac{\partial^2 f}{\partial x\partial y}(x,y) = \left(\frac{-1}{2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{3}{2}}\left(\frac{y}{b^2}\right)\cdot\left(\frac{2x}{a^2}\right) = \frac{-xy}{a^2b^2}\frac{1}{\sqrt{\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^3}}$$

$$\frac{\partial^2 f}{\partial y^2}(x,y) = \left(\frac{-1}{2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{3}{2}}\left(\frac{2y}{b^2}\right)\left(\frac{y}{b^2}\right) + \left(\frac{1}{b^2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{-\frac{1}{2}}$$

$$= -\frac{y^2}{b^4}\frac{1}{\sqrt{\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^3}} + \frac{1}{b^2}\frac{1}{\sqrt{\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)}}$$

(e) $f(x) = \ln(x^2+y^2+z+1)$

$$\frac{\partial f}{\partial x}(x,y,z) = \frac{1}{x^2+y^2+z+1}\cdot 2x \longrightarrow \frac{\partial^2 f}{\partial x^2}(x,y,z) = \frac{2(x^2+y^2+z+1) - 2x\cdot 2x}{(x^2+y^2+z+1)^2}$$

$$\frac{\partial^2 f}{\partial y\partial x}(x,y,z) = \frac{-2y}{(x^2+y^2+z+1)^2}(2x) = \frac{\partial^2 f}{\partial x\partial y}(x,y,z)$$

$$\frac{\partial^2 f}{\partial z\partial x}(x,y,z) = \frac{-2x}{(x^2+y^2+z+1)^2} = \frac{\partial^2 f}{\partial x\partial z}(x,y,z)$$

$$\frac{\partial^2 f}{\partial z\partial y}(x,y,z) = \frac{-1}{(x^2+y^2+z+1)^2}(2y)$$

$$\frac{\partial f}{\partial y}(x,y,z) = \frac{1}{x^2+y^2+z+1}2y \rightarrow \frac{\partial^2 f}{\partial y^2}(x,y,z) = \frac{2(x^2+y^2+z+1) - 2y\,2y}{(x^2+y^2+z+1)^2}$$

$$\frac{\partial f}{\partial z}(x,y,z) = \frac{1}{(x^2+y^2+z+1)}$$

$$\frac{\partial f}{\partial y\partial z}(x,y,z) = \frac{-2y}{(x^2+y^2+z+1)^2}$$

* Ejercicio 14 $f(x,y)=e^x \operatorname{sen}(y)$ → Demostrar $f \in C^2$ y $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$

- si $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$, entonces $\frac{\partial^2 f}{\partial x^2}=-\frac{\partial^2 f}{\partial y^2}$

$$\frac{\partial^2 f}{\partial y \partial x}(x,y)=e^x\cos(y)$$

- $\frac{\partial f}{\partial x}(x,y)=e^x\operatorname{sen}(y) \longrightarrow \frac{\partial^2 f}{\partial x^2}(x,y)=e^x\operatorname{sen}(y)$
- $\frac{\partial f}{\partial y}(x,y)=e^x\cos(y) \longrightarrow \frac{\partial^2 f}{\partial y^2}(x,y)=-e^x\operatorname{sen}(y)$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y)=e^x\cos(y)$$

$\longrightarrow \frac{\partial^2 f}{\partial x^2}=-\frac{\partial^2 f}{\partial y^2}$

⇒ f es de C^2 ya que existen todas las derivadas parciales de f hasta orden 2 y son continuas

$\longrightarrow$ f armónica

---

* Ejercicio 15 $f(x,y)=\begin{cases} 9-x^2-y^2 & \text{cuando } x^2+y^2\le 9 \\ 0 & \text{cuando } x^2+y^2>9 \end{cases}$

- Analizar cont y existencia de $\frac{\partial f}{\partial y}(3,0)$

⇒ Analizar continuidad en (3,0)

- $f(3,0)=9-3^2-0=0$
- $\lim_{(x,y)\to(3,0)} f(x,y)=\begin{cases} \lim_{(x,y)\to(3,0^+)} 0=0 \\ \lim_{(x,y)\to(3,0^-)} 9-x^2-y^2=0 \end{cases}$

$f(3,0)=\lim_{x,y\to 3,0} f(x,y)$ ⇒ f continua en (3,0)

⇒ Analizar existencia de derivada parcial

$$\frac{\partial f}{\partial y}(3,0)=\lim_{h\to 0}\frac{\overbrace{f(3,0+h)}^{>9}-\overbrace{f(3,0)}^{=9}}{h}=\lim_{h\to 0}\frac{\overbrace{0-[9-3^2-0]}^{\to 0}}{h}=0$$

$\frac{\partial f}{\partial y}(3,0)=0$

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* Ejercicio 16 Analizar existencia de derivadas direccionales en punto y dirección dados

ⓐ $f(x,y)=3x^2-2xy$, $P_0=(0,2)$ $\check{v}=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

$$P_0+h\check{v}=(0,2)+h\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)=\left(\frac{h}{2},2+\frac{\sqrt{3}}{2}h\right)$$

$$\frac{\partial f}{\partial \check{v}}(0,2)=\lim_{h\to 0}\frac{f\left(\frac{h}{2},\frac{2+h\sqrt{3}}{2}\right)-f(0,2)}{h}=\lim_{h\to 0}\frac{3\left(\frac{h}{2}\right)^2-2\left(\frac{h}{2}\right)\left(2+\frac{\sqrt{3}h}{2}\right)}{h}=\lim_{h\to 0}\left[\frac{3h^2}{4}-h\left(2-\frac{\sqrt{3}h}{2}\right)\right]\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\left[\frac{3h^2}{4}-2h+\frac{\sqrt{3}h^2}{2}\right]\cdot\frac{1}{h}=\lim_{h\to 0}\left[\frac{3h^2-8h+2\sqrt{3}h^2}{4}\right]\cdot\frac{1}{h}=\lim_{h\to 0}\frac{3h-8+2\sqrt{3}h}{4}=-2$$

$$\boxed{\frac{\partial f}{\partial \check{v}}(0,2)=-2}$$

(b) $f(x,y) = \begin{cases} \sqrt{xy} & \text{si } xy \geq 0 \\ x+y & \text{si } xy < 0 \end{cases}$ $P_0 = (0,0)$ $\check{v}_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\check{v}_2 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

⇒ Derivada direcc. f en P_0 y $\check{v}_1$

$$P_0 + h(\check{v}_1) = (0,0) + h\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left(\frac{h}{\sqrt{2}}, \frac{h}{\sqrt{2}}\right)$$

$$\frac{\partial f}{\partial \check{v}_1}(0,0) = \lim_{h\to 0} \frac{f\left(\frac{h}{\sqrt{2}}, \frac{h}{\sqrt{2}}\right) - f(0,0)}{h} = \lim_{h\to 0} \frac{\sqrt{\left(\frac{h}{\sqrt{2}}\right)^2}}{h} = \lim_{h\to 0} \frac{h}{\sqrt{2}} \cdot \frac{1}{h} = \frac{1}{\sqrt{2}}$$

⇒ Derivada direccional f en P_0 y $\check{v}_2$

$$P_0 + h(\check{v}_2) = (0,0) + h\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) = \left(\frac{h}{\sqrt{2}}, \frac{-h}{\sqrt{2}}\right)$$

$$\frac{\partial f}{\partial \check{v}_2}(0,0) = \lim_{h\to 0} \frac{f\left(\frac{h}{\sqrt{2}}, \frac{-h}{\sqrt{2}}\right) - \overset{\to 0}{f(0,0)}}{h} = \lim_{h\to 0} \frac{\overset{\to 0}{\frac{h}{\sqrt{2}} + \left(-\frac{h}{\sqrt{2}}\right)}}{h} = 0$$

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* Ejercicio 17 = Analizar exist derivadas direccionales en $P=(0,0)$ → ver si existe derivadas nulas → 0

↳ $\check{r} = (a,b)$, $\sqrt{a^2+b^2} = 1$

$$P + h\check{r} = (0,0) + h(a,b) = (ha, hb)$$

⇒ Relación $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$ con derivadas direccionales

↳ Podemos decir que gradiente $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$ es una transformación lineal → matriz de cambio de base

ya que cualquier derivada direccional sería una combinación lineal de las derivadas parciales → contenida en plano tg

↳ la deriv direcc. $D_v f$ es TL toma vector $\vec{v}$ y devuelve escalar tasa de cambio en esa dirección

↳ En $\mathbb{R}^2$. $D_v f = \nabla f \cdot v = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]\begin{pmatrix} a \\ b \end{pmatrix}$

(a) $f(x,y) = \frac{x+1}{x^2+y^2+1}$

$$\frac{\partial f}{\partial x}(x,y) = \frac{(1)(x^2+y^2+1) - (2x)(x+1)}{(x^2+y^2+1)^2} = \frac{x^2+y^2+1-2x^2-2x}{(x^2+y^2+1)^2} = \frac{y^2-x^2-2x+1}{(x^2+y^2+1)^2} \to \frac{\partial f}{\partial x}(0,0) = 1$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{(0)(x^2+y^2+1) - 2y(x+1)}{(x^2+y^2+1)^2} = \frac{-2y(x+1)}{(x^2+y^2+1)^2} \longrightarrow \frac{\partial f}{\partial y}(0,0) = 0$$

$\nabla f(0,0) = \left(\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial y}(0,0)\right)$

$\nabla f(0,0) = (1,0)$

• Vectores direccionales $\check{r} = (a,b)$

$$D_{\check{r}} f(0,0) = \nabla f(0,0) \cdot \check{r} = (1,0)\begin{bmatrix} a \\ b \end{bmatrix} = a + 0b = a$$

• Vectores direccionales nulas

$D_{\check{r}} f(0,0) = a = 0 \longrightarrow$ direcciones con derivada nula $\check{r}' = (0,1)$ v $\check{r}_2 = (0,-1)$

ⓑ $f(x,y) = \begin{cases} xy \ln(x^2+y^2) & \text{si } (x,y) \neq (0,0) \\ 0 & \text{si } (x,y) = (0,0) \end{cases}$

todas las direcciones existen y son nulas

$$\frac{\partial f}{\partial \vec{r}}(0,0) = \lim_{h\to 0} \left[ha\, hb \ln((ha)^2+(hb)^2) - 0 \right] \cdot \frac{1}{h} = \lim_{h\to 0} hab \ln(h^2(a^2+b^2)) = \lim_{h\to 0} hab\left[\overset{\to -\infty}{\ln(h^2)} + \underbrace{\ln(a^2+b^2)}_{0}\right] = 0$$

↳ Hallar gradiente

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{0 h \ln(h^2+0) - 0}{h} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0} \frac{0 h \ln(0+h^2) - 0}{h} = 0$$

$\nabla f(0,0) = (0,0)$

→ Derivada direccional

$D_{\vec{r}} f(0,0) = (0,0)\begin{pmatrix} a \\ b \end{pmatrix} = 0$ → todas las derivadas direccionales existen y son 0

ⓒ $f(x,y) = \sqrt{x^n + y^n}$, $n \in \mathbb{N} - \{1\}$

$$\frac{\partial f}{\partial \vec{r}}(0,0) = \lim_{h\to 0} \frac{\sqrt{(ha)^n + (hb)^n}}{h} = \lim_{h\to 0} \frac{\sqrt{h^n(a^n+b^n)}}{h} = \lim_{h\to 0} \frac{\sqrt{h^n}\,(a^n+b^n)}{h} = \lim_{h\to 0} \frac{n}{2} \cdot \frac{(a^n+b^n)}{\sqrt{h^n}} = ?$$

• hallar derivadas parciales

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{\sqrt{(h)^n}}{h} = \frac{\sqrt{h^n}}{h}$$

→ h^n tiene que ser positivo → $\frac{|h|^{\frac{n}{2}}}{h}$

$\begin{cases} \lim_{h\to 0} \frac{|h|}{h} & \text{si } n=2 \quad \text{no existe} \\ \lim_{h\to 0} h^{\frac{n}{2}-1} & \text{si } n\geq 3 \to 0 \end{cases}$

$\begin{cases} \lim_{h\to 0^+} \frac{|h|}{h} = 1 \\ \lim_{h\to 0^-} \frac{|h|}{h} = -1 \end{cases}$

$\frac{\partial f}{\partial x}(0,0) = 0$ si $n \geq 3$

→ Existen todas las derivadas direccionales y valen nulas para $n \geq 3$

ⓓ $f(x,y) = \begin{cases} \dfrac{e^{x^2+y^2}-1}{\sqrt{x^2+y^2}} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{si } (x,y) = (0,0) \end{cases}$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \left[\frac{e^{h^2+0^2}-1}{\sqrt{h^2+0^2}} - 0 \right] \cdot \frac{1}{h} = \lim_{h\to 0} \left[\frac{e^{h^2}-1}{\sqrt{h^2}} \right] \cdot \frac{1}{h} = \lim_{h\to 0} \frac{e^{h^2}-1}{|h|\, h} = \begin{cases} \lim_{h\to 0^+} \frac{e^{h^2}-1}{h^2} \overset{\text{L'H}}{=} \lim_{h\to 0^+} \frac{2he^{h^2}}{2h} = 1 \\ \lim_{h\to 0^-} \frac{e^{h^2}-1}{-h^2} = \lim_{h\to 0} \frac{2he^{h^2}}{2h} = -1 \end{cases}$$

$\frac{\partial f}{\partial y}(0,0) = 0$

$\nexists \frac{\partial f}{\partial x}(0,0)$ → $\nexists \frac{\partial f}{\partial y}(0,0)$ "misma simetría"

Como no existen derivadas parciales, no existen derivadas direccionales