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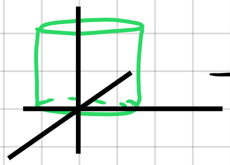
AREA DE UNA SUPERFICIE

$$\iint_D \| F'_u \times F'_v \| \, du\, dv$$

* Ejercicio 1: Calcular área de las superficies

ⓐ Σ: trozo sup. cilíndrica $x^2+y^2=4$, $0 \le z \le 2$

• Gráfico Σ



parametrizar superficie

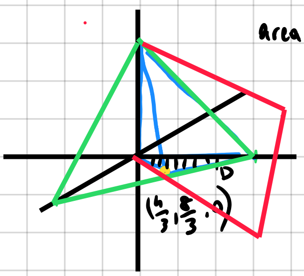
$$\vec{F}(\theta, z) = (2\cos(\theta), 2\operatorname{sen}(\theta), z), \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 2$$

$$\vec{N} = \frac{\partial \vec{F}}{\partial \theta} \times \frac{\partial \vec{F}}{\partial z} = \begin{vmatrix} i & j & k \\ -2\operatorname{sen}\theta & 2\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = (2\cos\theta, 2\operatorname{sen}\theta, 0)$$

$$\left\| \frac{\partial \vec{F}}{\partial \theta} \times \frac{\partial \vec{F}}{\partial z} \right\| = \sqrt{4\cos^2\theta + 4\operatorname{sen}^2\theta} = \sqrt{4} = 2$$

$$\text{área} = \int_0^2 \int_0^{2\pi} 2\, d\theta\, dz = \int_0^2 4\pi\, dz = 8\pi$$

ⓑ Σ: frontera del cuerpo definido por $x+y+z \le 4$, $y \ge 2x$ en 1° oct



$$\text{Area} = \text{area}(D_1) + \text{area}(D_2) + \text{area}(D_3) + \text{area}(D_4)$$

$$\underset{(\text{fondo})}{\text{area}(D_1)} = \int_0^4 \int_0^{4-y} dz\, dy = \int_0^4 4-y\, dy = 4y - \frac{y^2}{2} \Big|_0^4 = 16-8 = 8$$

$$\underset{(\text{abajo})}{\text{area}(D_2)} = \int_0^{4/3} \int_{2x}^{4-x} dy\, dx = \int_0^{4/3} 4-3x\, dx = 4x - \frac{3x^2}{2} \Big|_0^{4/3} = \frac{8}{3}$$

$x+y=4 \cap y=2x$

$$\text{area}(D_3) = \overbrace{\sqrt{\frac{4}{3}^2 + \frac{8}{3}^2}}^{\text{pitágoras}} \cdot \frac{4}{2} = \frac{8\sqrt{5}}{3}$$

$$\text{area}(D_4) = \text{parametrizar plano} \rightarrow \vec{F}(x,y) = (x, y, 4-x-y)$$

$$\frac{\partial F}{\partial x} \times \frac{\partial F}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = (1, 1, 1) \qquad \|\vec{N}\| = \sqrt{3}$$

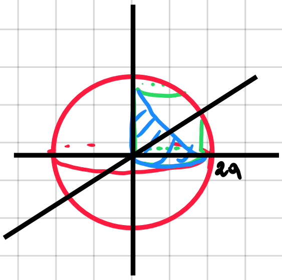
$$D_{xy} = \left\{ 0 \le x \le \frac{4}{3},\ 2x \le y \le 4-x \right\}$$

$$\text{area}(D_4) = \iint_{D_{xy}} \sqrt{3}\, dx\, dy = \sqrt{3} \int_0^{4/3} \int_{2x}^{4-x} dy\, dx = \sqrt{3} \cdot \frac{8}{3}$$

$$\text{area total} = 8 + \frac{8}{3} + \frac{8\sqrt{5}}{3} + \frac{8\sqrt{3}}{3} = 8\left(1 + \frac{1+\sqrt{5}+\sqrt{3}}{3}\right)$$

ⓒ Σ: trozo de superficie cilíndrica ec. $x^2+y^2=2ay$ interior de sup. $x^2+y^2+z^2 \le 4a^2$, 1° oct.

gráfico de Σ



• Analizar ecuaciones

$$x^2+y^2-2ay=0 \qquad x^2+y^2+z^2=4a^2$$

$$x^2+(y-a)^2=a^2 \qquad x^2+y^2+z^2=(2a)^2$$

∘ Hallar valor de z donde interseccionan

$$x^2+y^2+z^2=4a^2, \quad x^2+y^2=2ay$$

$$z^2+2ay=4a^2 \rightarrow z^2=4a^2-2ay=2a(2a-y) \rightarrow z=\sqrt{4a^2-2ay}$$

Superficie

$$\vec{F}(\theta,z)=(a\cos(\theta),\ a\,\mathrm{sen}(\theta)+a,\ z), \quad D=\{-\tfrac{\pi}{2}\le\theta\le\tfrac{\pi}{2},\ 0\le z\le\sqrt{4a^2-2ay}\}$$

$$\vec{N}=\frac{\partial\vec{F}}{\partial\theta}\times\frac{\partial\vec{F}}{\partial z}=\begin{vmatrix} i & j & k\\ a\,\mathrm{sen}\theta & a\cos & 0\\ 0 & 0 & 1\end{vmatrix}=(a\cos\theta,\ a\,\mathrm{sen}\theta,\ 0) \qquad \|\vec{N}\|=\sqrt{(a\cos\theta)^2+(a\,\mathrm{sen}\theta)^2}=a$$

$$\mathrm{area}(\Sigma)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\sqrt{4a^2-2ay}} a\,dz\,d\theta=a\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{4a^2-2ay}\,d\theta=a\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{4a^2-2a(a+a\,\mathrm{sen}\,\theta)}\,d\theta$$

$$\rightarrow \sqrt{4a^2-2a^2(1+\mathrm{sen}\theta)} \qquad \sqrt{2a^2(2-1-\mathrm{sen}\theta)} \qquad a\sqrt{2(1-\mathrm{sen}\theta)}$$

$$=a^2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{2(1-\mathrm{sen}\theta)}\,d\theta=\sqrt{2}a^2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{1-\mathrm{sen}\theta}\,d\theta=\left.\frac{2\sqrt{1-\mathrm{sen}(\theta)}\,\left(\mathrm{sen}\left(\frac{\theta}{2}\right)+\cos\left(\frac{\theta}{2}\right)\right)}{\cos\left(\frac{\theta}{2}\right)-\mathrm{sen}\left(\frac{\theta}{2}\right)}\right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\sqrt{2}a^2(2\sqrt{2})=4a^2$$

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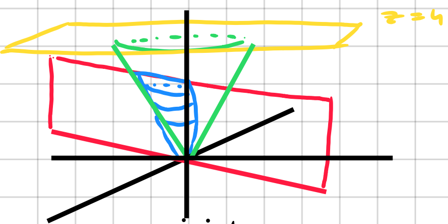
ⓓ Σ: trozo de sup. cónica $z=\sqrt{x^2+y^2}$ con $z\le 4$, $y\le\sqrt{3}\,x$

• Gráfico aprox Σ



$$\mathrm{area}(\Sigma)=\iint_D |F'_\mu \times F'_v|\,d\mu\,dv$$

• parametrización de superficie → fácil con coord cilíndricas

$$\begin{cases} x=r\cos(\theta)\\ y=r\,\mathrm{sen}(\theta)\\ z=z\end{cases} \qquad z=\sqrt{x^2+y^2}=\sqrt{r^2}=r$$

$$F(r,\theta)=(r\cos(\theta),\ r\,\mathrm{sen}(\theta),\ r)$$

$$F'_r\times F'_\theta=\begin{vmatrix} i & j & k\\ \cos & \mathrm{sen} & 1\\ -r\,\mathrm{sen} & r\cos & 0\end{vmatrix}=(-r\cos,\ -r\,\mathrm{sen},\ r)$$

$$\|N\|=\sqrt{r^2\cos^2+r^2\mathrm{sen}^2+r^2}=\sqrt{2r^2}=\sqrt{2}\,r$$

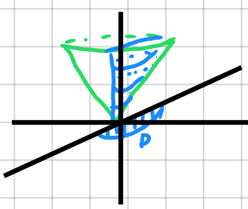
↳ lím integración $D=\{\underbrace{\sqrt{x^2+y^2}}_{r}\le z\le 4,\ y\le\sqrt{3}\,x\}$

$$r\,\mathrm{sen}(\theta)\le\sqrt{3}\,r\cos(\theta) \longrightarrow -\frac{5\pi}{6}\le\theta\le\frac{\pi}{3}$$

$$\longrightarrow 0\le r\le 4$$

$$\int_0^4\int_{-\frac{5\pi}{6}}^{\frac{\pi}{3}}\sqrt{2}\,r^2\,d\theta\,dr=\sqrt{2}\,r^4\,\frac{7\pi}{6}.$$

↳ calc área de mitad de un cono



$$\mathrm{Vol}=\iint_D dx\,dy\int dz$$

$$\mathrm{area}\,(\mathrm{perim})=\iint_D F(x,y)\,dx\,dy$$

INTEGRAL DE SUPERFICIE CAMPO ESCALAR

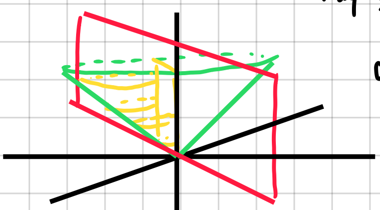
* **Ejercicio 3** calc masa porción sup cónica $4z^2 = x^2+y^2$, $0 \le z \le 1$, $x \le y$

densidad prop. punto al plano xy $\rightarrow$ $\delta(x,y,z) = k\,|z|$

- Gráfico fig análisis

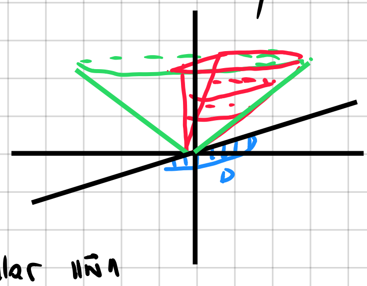
Hay que calcular la masa del cono (la frontera). como la densidad es la misma, puedo calcular lo mismo rotando la figura

altura plano xy



(original)

(rotación).



masa obj

$$\iint_\Sigma \delta\, ds = \iint_D \delta(\vec F)\cdot |\vec F_u' \times \vec F_v'|\, du\, dv$$

- parametrización sup con cilíndricas

$$\begin{cases} x = r\cos(\theta) \\ y = r\,\mathrm{sen}(\theta) \\ z = z \end{cases} \qquad 4z^2 = r^2(\cos^2\theta + \mathrm{sen}^2\theta)$$
$$2z = r \rightarrow z = \tfrac{1}{2}r$$

$$\vec F(r,\theta) = \left(r\cos(\theta),\ r\,\mathrm{sen}(\theta),\ \tfrac{1}{2}r\right), \quad \begin{cases} 0 \le \tfrac{1}{2}r \le 1 \rightarrow 0 \le r \le 2 \\ 0 \le \theta \le \pi \end{cases}$$

- Hallar $\|\vec N\|$

$$\vec N = \frac{\partial \vec F}{\partial r} \times \frac{\partial \vec F}{\partial \theta} = \begin{vmatrix} i & j & k \\ \cos & \mathrm{sen} & \tfrac{1}{2} \\ -r\,\mathrm{sen} & r\cos & 0 \end{vmatrix} = \left(-\tfrac{1}{2}r\cos(\theta),\ \tfrac{1}{2}r\,\mathrm{sen}(\theta),\ r\right)$$

$$\|\vec N\| = \sqrt{\left(\tfrac{1}{2}\right)^2\left[r^2(\cos^2+\mathrm{sen}^2)\right]+r^2} = \sqrt{r^2\left(\tfrac{1^2}{2}+1\right)} = r\sqrt{\tfrac{5}{4}}$$

- Hallar $\delta(\vec F(r,\theta))$

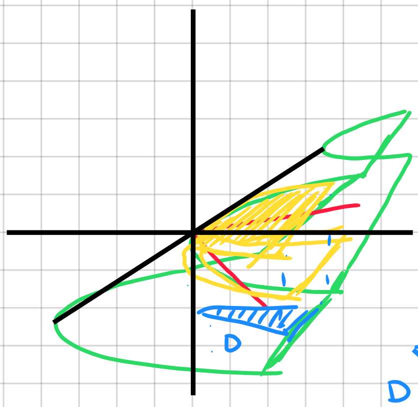
$$\delta(\vec F(r,\theta)) = k\,\left|\tfrac{1}{2}r\right| = \frac{kr}{2}$$

$$\text{masa} = \int_0^2\int_0^\pi \frac{kr}{2}\cdot r\sqrt{\frac{5}{4}}\, d\theta\, dr = \frac{k}{2}\cdot\frac{\sqrt5}{2}\int_0^2\int_0^\pi r^2\, d\theta\, dr = \frac{k\sqrt5}{4}\pi\int_0^2 r^2\, dr = \frac{k\sqrt5\,\pi}{4}\,\frac{r^3}{3}\Big|_0^2 = \frac{2\pi k\sqrt5}{3}$$

* *Ejercicio 5*: Calcule Integral de $f(x,y,z) = xy - z$ sobre sup. cilíndrica ee $y = z^2$ con $|x| \le y \le 2$

Gráfico sup cilíndrica

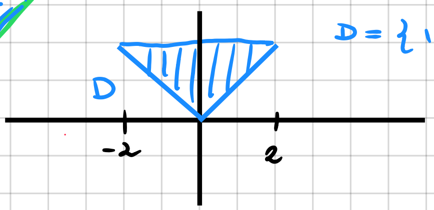
$$\iint_\Sigma f\, ds = \iint_D f(\vec{F}) \cdot |\vec{F}'_\mu \times F'_v|\, d\mu\, dv$$



• parametrizar superficie

$$\begin{cases} y = z^2 \\ |x| \le y \le 2 \end{cases} \qquad |x| \le z^2 \le 2$$

$$F(x,z) = (x, z^2, z)$$



$D = \{ |x| \le y \le 2 \}$

$x \le y \le 2$
$-x \le y \le 2$
$-2 \le x \le 2$

$\rightarrow |x| \le z^2 \le 2$

$\hookrightarrow -z^2 \le x \le z^2$

$\hookrightarrow 0 \le z^2 \le 2$
$-\sqrt{2} \le z \le \sqrt{2}$

• Hallar $\|\vec{N}\|$

$$\vec{N} = \frac{\partial F}{\partial x} \times \frac{\partial F}{\partial z} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 2z & 1 \end{vmatrix} = (0, -1, 2z) \rightarrow \|\vec{N}\| = \sqrt{1 + 4z^2}$$

• Hallar $f(\vec{F})$

$$f(F(x,z)) = xz^2 - z = z(xz - 1)$$

• Hallar integral

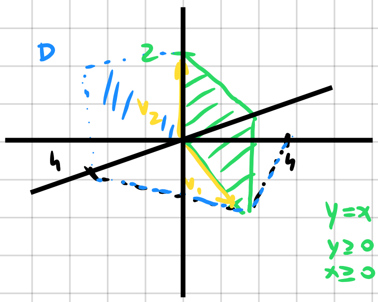
$$\iint_D f(\vec{F}) \cdot |F'_x \times F'_z|\, dx\,dz = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-z^2}^{z^2} (xz^2 - z)\sqrt{1+4z^2}\, dx\, dz$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{1+4z^2} \left(\frac{z^2 x^2}{2} - zx \right)\Bigg|_{-z^2}^{z^2} dz = \int_{-\sqrt{2}}^{\sqrt{2}} \underbrace{(-2z^3)\sqrt{1+4z^2}}_{\text{impar}}\, dz = 0$$

* Ejercicio 6 : Calcular valor medio f sobre sup plana $y=x$, $z\leq 2$, $y\leq 4$, 1° oct. $\begin{cases} x\geq 0 \\ y\geq 0 \\ z\geq 0 \end{cases}$

$f(x,y,z) = x^2yz$

• Gráfico sup.



$y\leq 4$
$x\leq 4$

$y=x$
$y\geq 0$
$x\geq 0$

$$\boxed{\text{valor medio } f = \frac{\text{total } f(\Sigma)}{\text{Area } (\Sigma)}}$$

$\text{area } \Sigma = \iint_D \|\vec{N}\|\, du\, dv$

$\text{total}_f(\Sigma) = \iint_D f(F) \cdot \|\vec{N}\|\, du\, dv$

• Hallar $\vec{F}$,

$\vec{F}(x,z) = (x, x, z)$

$D = \{ 0\leq x\leq 4 , 0\leq z\leq 2\}$

• Hallar $\|\vec{N}\|$

$\vec{N} = \frac{\partial\vec{F}}{\partial x} \times \frac{\partial\vec{F}}{\partial z} = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (1,-1,0)$ $\quad \|\vec{N}\| = \sqrt{2}$

• Hallar $f(\vec{F}(x,z))$

$f(\vec{F}(x,z)) = x^3 z$

• Hallar Area (Σ)

$\text{area}(\Sigma) = \iint_D \|\vec{N}\|\, du\, dv = \int_0^4 \int_0^2 \sqrt{2}\, dz\, dx = \int_0^4 2\sqrt{2}\, dx = 8\sqrt{2}$

• Hallar valor total $f(\Sigma)$

$\text{total } f(\Sigma) = \iint_D f(\vec{F})\, \|\vec{N}\|\, du\, dv = \int_0^4 \int_0^2 \sqrt{2}\, x^3 z\, dz\, dx = \sqrt{2}\int_0^4 x^3 \frac{z^2}{2}\Big|_0^2 = 2\sqrt{2}\int_0^4 x^3\, dx = 2\sqrt{2}\,\frac{x^4}{4}\Big|_0^4 = 128\sqrt{2}$

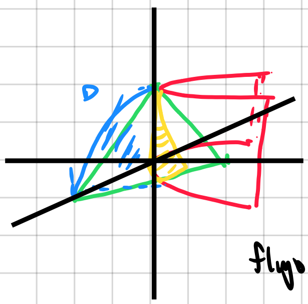
• Valor medio $f = \frac{128\sqrt{2}}{8\sqrt{2}} = \frac{128}{8} = 16$

Integral de superficie de campo vectorial

* Ejercicio 7 : Calcular flujo $\vec{f}$ a través de sup. Σ $\left(\iint_\Sigma \vec{f}\cdot\hat{n}\, d\sigma\right)$ indicando orientación (1° oct)

ⓐ $\vec{f}(x,y,z) = (y,\ x^2-y,\ xy)$ a través de trozo superficie cilíndrica ec $y=x^2$, $x+y+z\le 2$, $x\ge 0$, $y\ge 0$, $z\ge 0$

• Gráfico superficie



• Parametrizar curva

$F(x,z) = (x,\ x^2,\ z)$

$D_{xz} = \{ y=0 \wedge 0\le x\le 1 \wedge 0\le z\le 2-x^2-x \}$

Hallar D

$\begin{cases} y=x^2 \\ x+y+z\le 2 \end{cases}$ $\quad x^2+x+z\le 2$

$0\le z\le 2-x^2-x$

Hallar intersección $y=x^2$ y $x+y+z=2$ donde $z=0$

$x+x^2=2 \longrightarrow x^2+x-2=0$

$\frac{-1\pm\sqrt{1^2-4(-2)}}{2} = \boxed{x_1=1}$, $x_2=-2$ $\longrightarrow 0\le x\le 1$

$\text{flujo}_f(\Sigma) = \iint_{D_{xz}} \vec{f}(\vec{F}(x,z))\cdot(F'_x\times F'_z)\,dx\,dz$

• Hallar $\vec{N}$

$\vec{N} = \frac{\partial\vec{F}}{\partial x}\times\frac{\partial\vec{F}}{\partial z} = \begin{vmatrix} i & j & k \\ 1 & 2x & 0 \\ 0 & 0 & 1 \end{vmatrix} = (2x,\ -1,\ 0)$

• Hallar $\vec{f}(\vec{F}(x,z)) = \vec{f}(x, x^2, z)$

$\vec{f}(x,x^2,z) = (x^2,\ 0,\ x^3)$

• Calcular flujo

$\text{flujo}_f(\Sigma) = \iint_{D_{xz}} \vec{f}(\vec{F}(x,z))\cdot(F'_x\times F'_z)\,dx\,dz = \int_0^1\int_0^{2-x^2-x} (2x,-1,0)\,(x^2,0,x^3)\,dz\,dx = \int_0^1\int_0^{2-x^2-x} 2x^3\,dz\,dx$

$\int_0^{2-x^2-x} 2x^3\,dz = 2x^3(2-x^2-x) = 4x^3-2x^5-2x^4$

$\int_0^1 4x^3-2x^5-2x^4\,dx = \frac{4x^4}{4}-\frac{2x^6}{6}-\frac{2x^5}{5}\Big|_0^1 = x^4-\frac{1}{3}x^6-\frac{2}{5}x^5\Big|_0^1 = \frac{4}{15}$

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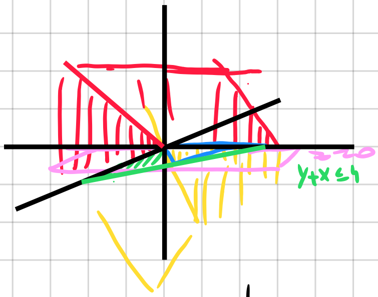
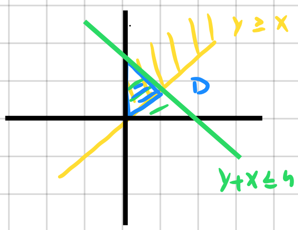
(b) $\vec{f}(x,y,z) = (x-y,\ x,\ 2-y)$ a través de sup frontera def por: Σ

$x+y\le 4,\quad y\ge x,\quad z\le x,\quad z\ge 0$

• Gráfico aprox superf.



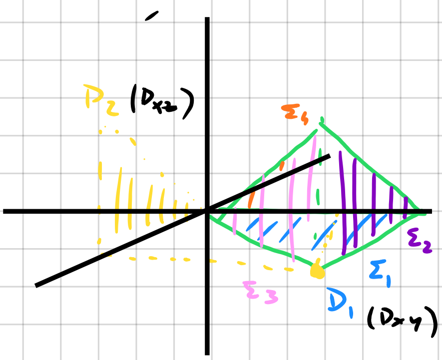
Visualización z=0



$$\iint_\Sigma \vec f\,\vec n\,ds \to \iint_{\Sigma_1}\vec f\,\vec n\,ds + \dots + \int_{\Sigma_4}\vec f\,\vec n\,ds$$

↓

4 lados $\Sigma_1,\ \Sigma_2\ \Sigma_3\ \Sigma_4$



• Hallar flujo en Σ_1, $\Sigma_1 = \boxed{z=0}$

$F_1(x,y) = (x,y,0)$

$$\frac{\partial F_1}{\partial x}\times\frac{\partial F}{\partial y} = \begin{vmatrix} i&j&k\\1&0&0\\0&1&0\end{vmatrix} = (0,0,1) \to (0,0,-1)$$

$f(F_1(x,y)) = (xy,\ x,\ 2-y)$, $D=\{0\le x\le 2,\ x\le y\le 4-x\}$

$$\text{flujo}(\Sigma_1) = \int_0^2\int_x^{4-x}(xy,x,2-y)(0,0,-1)\,dy\,dx = \int_0^2 \left(\frac{y^2}{2}-2y\right)\Big|_x^{4-x} dx =$$

$$\int_0^2 \left[\frac{(4-x)^2}{2} - 2(4-x)\right] - \left(\frac{x^2}{2}-2x\right) dx$$

$$= \int_0^2 8 - 4x + \frac{x^2}{2} - 8 + 2x - \frac{x^2}{2} + 2x\,dx = \int_0^2 0\,dx = 0$$

o Hallar flujo $\Sigma_4 \to 0\le z\le 0$

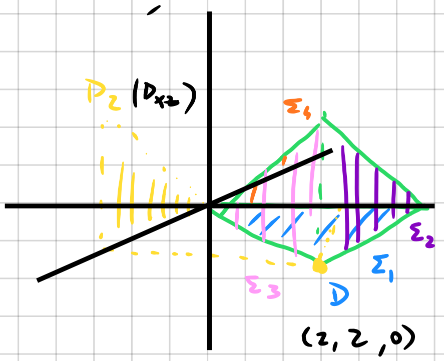
$F(x,y) = (x,y,x)$ $\qquad D=\{0\le x\le 2,\ x\le y\le 4-x\}$

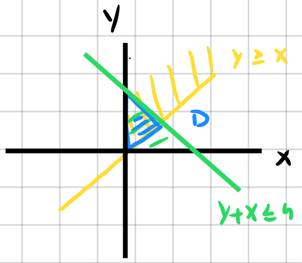
$f(x,y,x) = (x-y,\ x,\ 2-y)$ $\qquad \frac{\partial \vec F}{\partial x}\times\frac{\partial \vec F}{\partial y} = \begin{vmatrix} i&j&k\\1&0&1\\0&1&0\end{vmatrix} = (-1,0,1)$

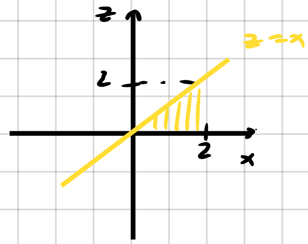
$$\text{flujo } \Sigma_4 = \int_0^2\int_x^{4-x}(x-y,x,2-y)(-1,0,1)\,dy\,dx = \int_0^2\int_x^{4-x} y-x+2-y\,dy\,dx = \int_0^2 -xy+2y\Big|_x^{4-x} = \int_0^2 \left[x(4-x)+2(4-x)\right]$$

$$= \int_0^2 \left[x(4-x)+2(4-x)\right] - \left[-x^2+2x\right] dx = \int_0^2 x^2-4x-2x+8+x^2-2x\,dx = \int_0^2 2x^2-8x+8\,dx$$

$$= \frac{2x^3}{3} - 4x^2 + 8x\Big|_0^2 = \frac{16}{3}$$







• Hallar flujo Σ_2. $\quad y + x \leq 4 \rightarrow y = 4 - x$

$$F(x,z) = (x,\ 4-x,\ z) \qquad D_2 = \{ 0 \leq x \leq 2,\ 0 \leq z \leq x \}$$

$$\frac{\partial F}{\partial x} \times \frac{\partial F}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (-1, -1, 0) \longrightarrow (1, 1, 0) \text{ orientación afuera}$$

$$f(x, 4-x, z) = (2x-4,\ x,\ x-2)$$

$$\text{flujo}(\Sigma_2) = \int_0^2 \int_0^x (2x-4, x, x-2)(1,1,0)\, dz\, dx = \int_0^2 \int_0^x 2x - 4 + x \, dz\, dx = \int_0^2 3xz - 4z \Big|_0^x dx =$$

$$\int_0^2 3x^2 - 4x\, dx = x^3 - 2x^2 \Big|_0^2 = 0$$

• Hallar flujo Σ_3. $\quad y \geq x \rightarrow y = x$

$$F(x,z) = (x,\ x,\ z) \quad - \quad D = \{ 0 \leq x \leq 2,\ 0 \leq z \leq x \}$$

$$\frac{\partial F}{\partial x} \times \frac{\partial F}{\partial z} = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (1, -1, 0)$$

$$f((x,x,z)) = (0,\ x,\ 2-x)$$

$$\text{flujo}(\Sigma_3) = \int_0^2 \int_0^x (0, x, 2-x)(1,-1,0)\, dz\, dx = \int_0^2 \int_0^x -x\, dz\, dx = \int_0^2 -x^2\, dx = -\frac{x^3}{3}\Big|_0^2 = -\frac{8}{3}$$

$$\oiint_{\partial E} \vec{F}\, d\vec{S} = \frac{16}{3} - \frac{8}{3} = \frac{8}{3}$$

ⓒ $\vec{f}(x,y,z) = (y^3 z,\ xz - yz,\ x^2 z)$, $\underset{\ge 0}{2y} = \underset{\ge 0}{x^2}$, $\sqrt{2y} = x$ $\qquad x^2+y^2+z^2 \le 3$, 1° oct $\begin{cases} x \ge 0 \\ y \ge 0 \\ z \ge 0 \end{cases}$

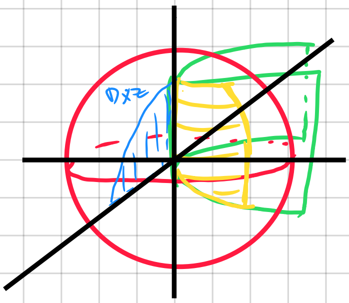
• <u>Gráfico aprox</u> sup.

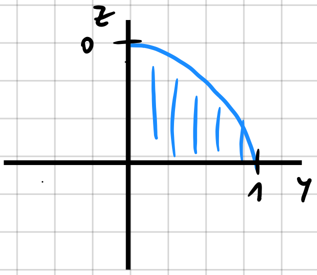
parametrizar superficie

$F(x,y) = (\sqrt{2y},\ y\ ,z)$

$\frac{\partial F}{\partial y} \times \frac{\partial F}{\partial z} = \begin{vmatrix} i & j & k \\ \frac{1}{\sqrt{2y}} & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (1,\ -\frac{1}{\sqrt{2y}},\ 0)$

$f(\sqrt{2y}, y, z) = (y^3 z,\ \sqrt{2y}\, z - yz,\ 2yz)$





• Hallar Dxz → intersección de parábola con esfera primer oct

$\begin{cases} x^2+y^2+z^2 \le 3 \\ 2y = x^2 \end{cases}$ $\qquad y^2+2y+z^2 \le 3$

$0 \le z \le \underbrace{\sqrt{3-y^2-2y}}_{\ge 0}$ → $\sqrt{3-y^2-2y} = 0$

$\frac{2 \pm \sqrt{2^2-4(-1)(3)}}{-2} = \begin{cases} y_1 = 1 \\ y_2 = -3 \end{cases}$

$D_{xz} = \{\ 0 \le y \le 1,\ \ 0 \le z \le \sqrt{3-y^2-2y}\ \}$

$$\int_0^1 \int_0^{\sqrt{2+(y+1)^2}} (y^3 z, \sqrt{2y}\, z - yz, 2yz)\left(1, \frac{-1}{\sqrt{2y}}, 0\right) dz\, dy = \int_0^1 \int_0^{\sqrt{2+(y+1)^2}} y^3 z + \left(-z + \frac{yz}{\sqrt{2y}}\right) dz\, dy$$

• Hallar Dxz → intersección de parábola con esfera primer oct

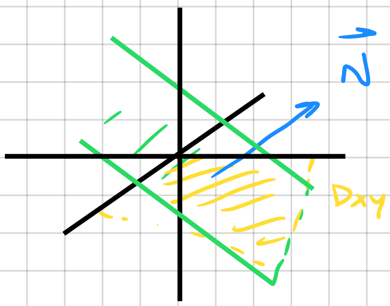
$\begin{cases} x^2+y^2+z^2 \le 3 \\ 2y = x^2 \end{cases}$ $\qquad y^2+2y+z^2 \le 3$

$(y+1)^2 + z^2 \le 4$ → $\begin{cases} y = r\cos(\theta) \\ z = r\,\text{sen}(\theta) \end{cases}$, $0 \le r \le 2$

seguir resolviendo...

∗ Ejercicio 10 : $f(x,y,z) = (x-y,\ az,\ by)$, Hallar rel a/b para $\vec{f}$ nulo en ec $y+z=3$, $1°$ oct, $x \le 2$

• Gráfico aprox



• parametrizar plano

$$F(x,y) = (x,\ y,\ 3-y)$$

• Hallar $\frac{\partial F}{\partial x} \times \frac{\partial F}{\partial y}$

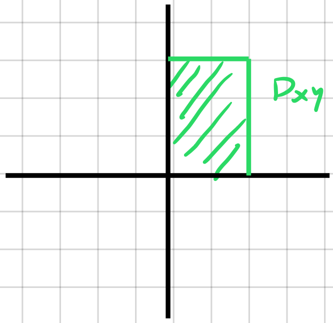
$$\frac{\partial F}{\partial x} \times \frac{\partial F}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = (0,\ 1,\ 1)$$

• Hallar D_{xy}

D_{xy} $\{ 0 \le x \le 2,\quad 0 \le y \le 3 \}$

y se obtiene por intersección

$y+z=3$ con $z=0$
$y=3$



• Hallar $\vec{f}(\vec{F}(x,y))$

$$\vec{f}(\vec{F}(x,y)) = (x-y,\ a(3-y),\ by)$$

• Hallar a,b para flujo nulo

$$\iint_{D_{xy}} (x-y,\ a(3-y),\ by)(0,1,1)\, dx\,dy = \iint_{D_{xy}} 3a - ay + by \; dx\,dy$$

$$= \int_0^2 \int_0^3 3a - ay + by \; dy\,dx = \int_0^2 3ay - \frac{ay^2}{2} + \frac{by^2}{2} \Big|_0^3 dx$$

$$= \int_0^2 9a - \frac{81}{2} + \frac{9b}{2}\, dx = 9ax - \frac{81}{2}x + \frac{9b}{2}x \Big|_0^2 = 18a - 81 + 9b$$

Hallar a,b . flujo nulo

flujo $= 18a - 81 + 9b = 0$

$9(9a+b) = 81$
$9a+b = 9$
$a = \frac{9-b}{9} = 1 - \frac{b}{9}$

$$\int_0^2 9a + \frac{81}{2}(b-a)\, dx = 0$$

$$9ax + \frac{81}{2}x\,(b-a)\Big|_0^2$$

?

$= 18a + 81(b-a) = 0$

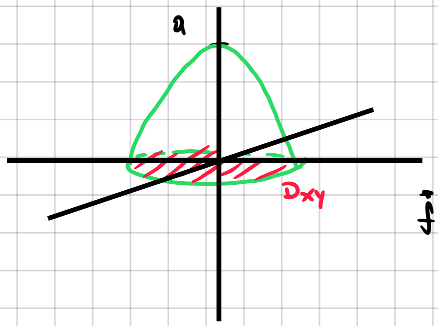
$18a + 81b - 81a = 0$

* Ejercicio 11 = $\vec{f}(x,y,z) = (x,y,z)$,

Calcular flujo $\vec{f}$ por sup. $z = a - x^2 - y^2 = a - (x^2+y^2)$, encima $z=0$, $a>0$

Considerar normal $\check{n}$ componente z pos

- Figura a analizar



- Hallar parametrización

$F(x,y) = (x, y, a-(x^2+y^2))$

$\frac{\partial F}{\partial x} \times \frac{\partial F}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & 0 & -2x \\ 0 & 1 & -2y \end{vmatrix} = (2x, 2y, 1)$

$\vec{f}(\vec{F}(x,y)) = \vec{f}(x, y, a-(x^2+y^2))$

$= (x, y, a-(x^2+y^2))$

- Hallar D_{xy}

Intersección $z = a - x^2 - y^2$ con $z=0$

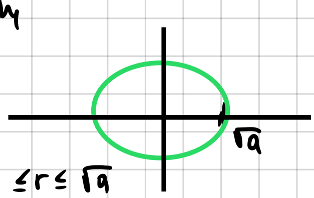
$\boxed{x^2+y^2=a}$

$D_{xy}\ \{(x,y)\in\mathbb{R}^2 / x^2+y^2=a\}$

flujo $= \iint_{D_{xy}} (x, y, a-(x^2+y^2))(2x, 2y, 1)\, dxdy = \iint_{D_{xy}} 2x^2+2y^2+a-x^2-y^2\, dxdy$

$= \iint_{D_{xy}} x^2+y^2+a\, dxdy = \int_0^{\sqrt{a}}\int_0^{2\pi} (r^2+a).r\, d\theta dr$

$D_{xy} = \{x^2+y^2=a\}$



$\begin{cases} x = r\cos(\theta) \\ y = r\,\mathrm{sen}(\theta) \end{cases}$, $0 \le r \le \sqrt{a}$, $0 < \theta \le 2\pi$, $|J(r,\theta)| = r$

$= 2\pi \int_0^{\sqrt{a}} r^3 + ar\, dr = 2\pi\left(\frac{r^4}{4} + \frac{ar^2}{2}\Big|_0^{\sqrt{a}}\right) = 2\pi\left(\frac{a^2}{4}+\frac{a^2}{2}\right) = \frac{3\pi a^2}{2}$

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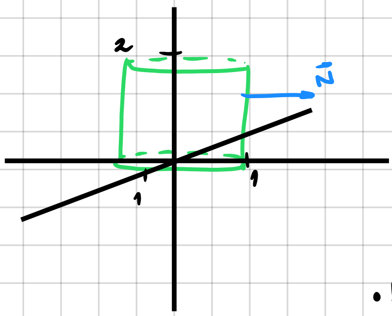
* Ejercicio 13 : $\vec{f}(x,y,z) = (4y, y, \varphi)$, calcular flujo $\vec{f}$ através sup. cilíndrica $\begin{cases} x^2+y^2=1 \\ 0 \le z \le 2 \end{cases}$

Gráfico superficie



- Hallar parametrización de superficie

$\vec{\sigma}(\theta, z) = (\cos(\theta), \mathrm{sen}(\theta), z)$

$D = \begin{cases} x^2+y^2=1 \\ 0 \le z \le 2 \end{cases} \longrightarrow \boxed{r=1}$, $0 \le \theta \le 2\pi$

- Hallar orientación

$\frac{\partial\vec{\sigma}}{\partial\theta} \times \frac{\partial\vec{\sigma}}{\partial z} = \begin{vmatrix} i & j & k \\ -\mathrm{sen}(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\cos(\theta), \mathrm{sen}(\theta), 0)$

- Hallar $\vec{f}(\vec{\sigma}(\theta, z))$

$\vec{f}(\vec{\sigma}(\theta,z)) = (4\,\mathrm{sen}(\theta), \mathrm{sen}(\theta), \varphi)$

- Hallar flujo

$\iint_D (4\,\mathrm{sen}, \mathrm{sen}, \varphi)\cdot(\cos, \mathrm{sen}, 0)\, dz d\theta = \int_0^2\int_0^{2\pi} 2\,\mathrm{sen}(2\theta) + \mathrm{sen}^2(\theta)\, d\theta dz$

$\hookrightarrow \mathrm{sen}^2(\theta) = 1 - \cos^2(\theta)$

$\mathrm{sen}^2(\theta) = \frac{1}{2} - \frac{\cos(2\theta)}{2}$

$\mathrm{sen}^2+\cos^2 = 1$
$\cos^2 - \mathrm{sen}^2 = \cos(2x)$
$2\cos^2 = \cos(2x)+1$
$\cos^2 = \frac{\cos(2x)+1}{2}$

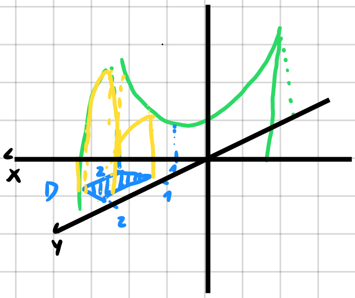
$\int_0^2\int_0^{2\pi} 2\,\mathrm{sen}(2\theta) + \frac{1}{2} - \frac{\cos(2\theta)}{2}\, d\theta dz$

$\int_0^2 -\cos(2\theta) + \frac{1}{2}\theta - \frac{\mathrm{sen}(2\theta)}{2}\Big|_0^{2\pi} dz = \int_0^2 (-1+\pi)-(-1)\, dz = \int_0^2 \pi\, dz = 2\pi$

* Ejercicio 14: $\vec{f}(x,y,z) = (abx,\ y/a,\ -z/b)$ Hallar a y b para existencia mín del flujo $\vec{f}$

a través Σ: $z = xy$, $(x,y) \in [1,2]\times[1,2]$ $\underbrace{\tilde{n}\cdot(0,0,1) < 0}$ → comp. $z < 0$

• Gráfico Sup



• parametrizar sup

$\vec{F}(x,y) = (x,\ y,\ xy)$ $D = \{1 \leq x \leq 2,\ 1 \leq y \leq 2\}$

$$\frac{\partial \vec{F}}{\partial x} \times \frac{\partial \vec{F}}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & 0 & y \\ 0 & 1 & x \end{vmatrix} = (-y,\ -x,\ 1) \rightarrow (y,\ x,\ -1)$$

$\tilde{n}\cdot(0,0,1) < 0$

• Hallar $\vec{f}(\vec{F})$

$$\vec{f}(\vec{F}(x,y)) = (abx,\ y/a,\ -xy/b)$$

• calcular flujo

$$\iint_D (abx,\ y/a,\ -xy/b)(y,\ x,\ -1)\,dx\,dy = \iint_D abxy + yx/a + xy/b\ dx\,dy = \iint_D xy\left(ab + \frac{1}{a} + \frac{1}{b}\right)dx\,dy$$

$$= \left(ab + \frac{1}{a} + \frac{1}{b}\right)\int_1^2\int_1^2 xy\,dy\,dx = \left(ab+\frac{1}{a}+\frac{1}{b}\right)\int_1^2 \frac{xy^2}{2}\Big|_1^2 dx = \left(ab+\frac{1}{a}+\frac{1}{b}\right)\int_1^2 \frac{3x}{2}\,dx = \left(ab+\frac{1}{a}+\frac{1}{b}\right)\frac{3x^2}{4}\Big|_1^2$$

$$= \left(ab+\frac{1}{a}+\frac{1}{b}\right)\left(3 - \frac{3}{4}\right) = \frac{9}{4}\left(ab + \frac{1}{a} + \frac{1}{b}\right)$$

• Hallar a,b relativo mín flujo $g(a,b) = \frac{9}{4}ab + \frac{9}{4}\cdot a^{-1} + \frac{9}{4}\cdot b^{-1}$

$$\frac{\partial g}{\partial a} = \frac{9}{4}b - \frac{9}{4}a^{-2}, \quad \frac{\partial g}{\partial b} = \frac{9}{4}a - \frac{9}{4}\cdot b^{-2}$$

• Hallar (a,b) críticos

$$\begin{cases} \frac{9}{4}b - \frac{9}{4}\cdot\frac{1}{a^2} = 0 \\ \frac{9}{4}a - \frac{9}{4}\frac{1}{b^2} = 0 \end{cases} \rightarrow \frac{9}{4}b - \frac{9}{4}\frac{1}{a^2} = \frac{9}{4}a - \frac{9}{4}\frac{1}{b^2}$$

$$b - \frac{1}{a^2} = a - \frac{1}{b^2}$$

$$b + \frac{1}{b^2} = a + \frac{1}{a^2}$$

puntos donde $\boxed{b = a}$

$(a,a) \rightarrow (b,b)$

• Hessiana

$$Hg(a,b) = \begin{vmatrix} +\frac{9}{2}a^{-3} & \frac{9}{4} \\ \frac{9}{4} & \frac{9}{2}b^{-3} \end{vmatrix}$$

$\frac{9}{2}\frac{1}{a^3} \geq 0 \rightarrow a \geq 0$

$$Hg(a,a) = \begin{vmatrix} \frac{9}{2}a^{-3} & \frac{9}{4} \\ \frac{9}{4} & \frac{9}{2}a^{-3} \end{vmatrix} = \frac{81}{4}a^{-6} - \frac{81}{16} = \frac{81}{4}\left(\underbrace{\frac{1}{a^6} - \frac{1}{4}}\right)$$

$\frac{1}{a^6} - \frac{1}{4} > 0$

$\frac{1}{a^6} \geq \frac{1}{4}$ $a^6 < 4$

$a < \sqrt[6]{4}$

puntos (a,a) donde $a \leq \sqrt[6]{4}$