20/05

Resolución Guía 7

Parametrización y orientación de curvas

* **Ejercicio 2** $\quad g = [0, 2] \to \mathbb{R}^2 \;/\; \vec{g}(t) = (\cos(t), \operatorname{sen}(t))$

(a) Demostrar rapidez constante

$$\| g'(t) \| = \sqrt{\cos^2(t) + \operatorname{sen}^2(t)} = 1$$

(b) Reparametrizar curva 4 veces más rápido

$$g(\mu) = (\cos(4\mu), \operatorname{sen}(4\mu))$$

acotar intervalo

$0 \leq t \leq 2\pi$

$0 \leq \mu \leq \frac{\pi}{2}$

$$g'(\mu) = (-4\operatorname{sen}(4\mu), 4\cos(4\mu))$$

$$\| g'(\mu) \| = \sqrt{(4\cos(4\mu))^2 + (4\operatorname{sen}(4\mu))^2} = \sqrt{4^2} = 4 = 4 \cdot \|g'(t)\|$$

$\Rightarrow$ Hallar aceleración

$$g''(\mu) = (-16\cos(4\mu), -16\operatorname{sen}(4\mu))$$

$\Rightarrow$ Hallar μ cuando pasa por $(0,1) = g(\mu)$ $\rightarrow \mu = \pi/8$

$$g'(\pi/8) = (-4, 0) \to \text{velocidad}$$

$$g''(\pi/8) = (0, -16) \to \text{aceleración}$$

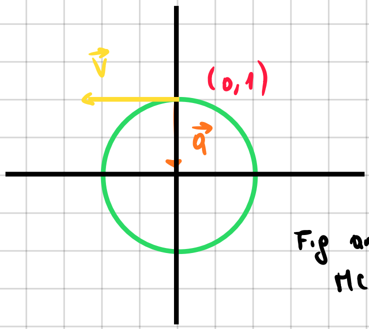


Fig análisis MCU

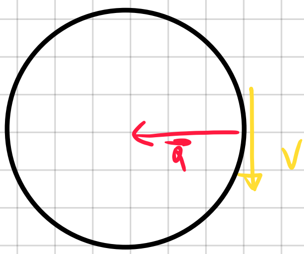
(c) reparametrizar curva 2 veces más lento

$$g(\mu) = \left(\cos\left(-\frac{\mu}{2}\right), \operatorname{sen}\left(-\frac{\mu}{2}\right)\right)$$

$\mu \in [0, 4\pi]$

$$g'(\mu) = \left(\frac{1}{2}\operatorname{sen}(\mu/2), -\frac{1}{2}\cos(\mu/2)\right)$$

$$\| g'(\mu) \| = \sqrt{\left(\frac{1}{2}\right)\left(\operatorname{sen}^2(\mu/2) + \cos^2(\mu/2)\right)} = \frac{1}{2} \to \text{rapidez} = \frac{1}{2}\|g'(t)\|$$



Integral de campos escalares a lo largo de curvas

* Ejercicio 3: Calcular $\int_C f\, ds = \int_{t_0}^{t} f(\rho(t))\, \|\rho'(t)\|\, dt$

ⓐ $f(x,y) = 1/(x^2+y^2)$ $\quad C: x^2+y^2=4,\ y \geq 0$ $\curvearrowright$ 1er y 2do cuadrante

↳ Hallar parametrización $C = \bar{\rho}(t) = (2\cos(t), 2\operatorname{sen}(t)),\ t \in [0, \pi]$

↳ Hallar $f(\rho(t))$

$$\bar{f}(\bar{\rho}(t)) = \frac{1}{(2\cos(t))^2 + (2\operatorname{sen}(t))^2} = \frac{1}{4}$$

↳ Hallar $\|\rho'(t)\|$

$$\rho'(t) = (-2\operatorname{sen}(t), 2\cos(t)) \Rightarrow \|\rho'(t)\| = \sqrt{(2\cos(t))^2 + (2\operatorname{sen}(t))^2} = 2$$

↳ $\oint_C f\, ds$

$$\oint_C f\, ds = \int_0^{\pi} \frac{1}{2}\, dt = \frac{\pi}{2}$$

ⓑ $f(x,y,z) = 2x - yz$ $\quad C$ recta intersección $\pi_1 \cap \pi_2 \,/\, \pi_1 = 2y - x + z = 2,\ \pi_2 = x - y + z = 4$ desde $a = (7,4,1)$ $b = (4,2,2)$

⇒ Hallar $r(t)$ intersección entre $\pi_1 \cap \pi_2$

↳ $n_1 = (-1,2,1)$
↳ $n_2 = (1,-1,1)$

$$\vec{v} = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ 1 & -1 & 1 \end{vmatrix} = (3, 2, -1)$$

recta $r(t) = t(3,2,-1) + (7,4,1) = (3t+7,\ 2t+4,\ -t+1)$

⇒ Hallar Intervalo de t

$r(0) = 0.(3,2,-1) + (7,4,1) = (7,4,1)$

$r(t) = t(3,2,-1) + (7,4,1) = (4,2,2)$

$$\begin{cases} 3t+7 = 4 \to 3t = -3 \\ 2t+4 = 2 \to 2t = -2 \to \boxed{t=-1} \\ -t+1 = 2 \to -t = 1 \end{cases}$$

⇒ Hallar $f(\vec{r}(t))$

$$f(\vec{r}(t)) = 2(3t+7) - (2t+4)(-t+1)$$
$$= (6t+14) - (-2t^2+2t-4t+4)$$
$$= 6t+14+2t^2-2t+4t-4$$
$$= 2t^2+8t+10$$

⇒ Hallar $\|\rho'(t)\|$

$\rho'(t) = (3,2,-1)$ $\quad \|\rho'(t)\| = \sqrt{3^2+2^2+1^2} = \sqrt{14}$

⇒ Hallar $\oint_C f\, ds$

$$\oint_C f\, ds = \int_0^{-1} (2t^2+8t+10)\sqrt{14}\, dt = \sqrt{14}\left(2\int_0^{-1} t^2\, dt + \int_0^{-1} 8t\, dt + \int_0^{-1} 10\, dt\right)$$
$$= \sqrt{14}\left(\frac{2}{3}t^3 + \frac{8}{2}t^2 + 10t\right)$$
$$= \sqrt{14}\left(-\frac{2}{3} + 4 - 10\right) = -\frac{20\sqrt{14}}{3}$$

* Ejercicio 4 Calcular longitud

(a) Curva $\vec{\sigma}(t) = \left(t, \frac{2}{3}t^{\frac{3}{2}}, 2\right)$, $t \in [0,3]$

$$\text{long}(c) = \int_0^3 \|\sigma'(t)\| dt = \int_0^3 (1+t)^{\frac{1}{2}} dt = \frac{2}{3}(1+t)^{3/2}\Big|_0^3 = {}^{14}/_3$$

$$\sigma'(t) = \left(1, t^{\frac{1}{2}}, 0\right) = (1, \sqrt{t}, 0) \longrightarrow \|\sigma'(t)\| = \sqrt{1+\sqrt{t}^2} = \sqrt{1+t}$$

$$\hookrightarrow s(t) = \int_0^t \|\sigma'(\mu)\| d\mu = \frac{2}{3}(1+t)^{\frac{3}{2}} - \frac{2}{3}$$

(b) Hélice $\vec{x} = (3\cos(t), 3\,\text{sen}(t), 4t)$ $t \in [0, 2\pi]$

$$\text{Long}(c) = \int_0^{2\pi} \|g'(t)\| dt = \int_0^{2\pi} 5\, dt = 10\pi$$

$$\vec{g}'(t) = (-3\,\text{sen}(t), 3\cos(t), 4) \qquad \|g'(t)\| = \sqrt{3^2(\text{sen}^2 t + \cos^2 t) + 4^2} = \sqrt{25} = 5$$

$\Rightarrow$ reparam $s(t) = \int_0^t \|g'(\mu)\| d\mu = 5\mu\Big|_0^t = 5t \longrightarrow s(2\pi) = 10\pi$

* Ejercicio 5 Intersección $\Sigma_1 = z = 2 - x^2 - 2y^2$ $\Sigma_2 : z = x^2$

$\Rightarrow$ Hallar curva de intersección

$$\begin{cases} z = 2 - x^2 - 2y^2 \\ z = x^2 \end{cases} \longrightarrow \begin{array}{l} z = 2 - z - 2y^2 \\ 2z = 2 - 2y^2 \end{array} \quad \boxed{z = 1 - y^2} \quad \boxed{z = x^2}$$

$$x^2 = 1 - y^2$$

$$\boxed{y^2 + x^2 = 1} \longrightarrow \text{curva } (\cos(t), \text{sen}(t)) \quad \boxed{z = \cos(t)^2}$$

$$\hookrightarrow x = \sqrt{1-y^2}$$
$$= -\sqrt{1-y^2}$$

$$g(t) = \left(\sqrt{1-t^2}, t, 1-t^2\right) \checkmark$$

$$g(t) = (\cos(t), \text{sen}(t), \cos^2(t))$$

$\rightarrow$ Hallar $\|g'(t)\|$

$$g'(t) = \left(\frac{1}{2\sqrt{1-t^2}}(-2t), 1, -2t\right)$$

$$\|g'(t)\| = \sqrt{\frac{t^2}{(1-t^2)} + 1 + 4t^2} = \sqrt{\frac{1}{(1-t^2)} + 4t^2}$$

$\rightarrow$ Hallar $\delta(g(t))$

$$\delta(g(t)) = k\left(\sqrt{1-t^2} \cdot t\right)$$

$\rightarrow$ Hallar masa

$$M = \int_0^1 \delta(g(t)) \|g'(t)\| dt$$

$$\int k\left(\sqrt{1-t^2} \cdot t\right) \sqrt{\frac{1}{(1-t^2)} + 4t^2}\, dt$$

$$= k\int t\left(\sqrt{1-t^2}\right) \frac{1}{\sqrt{1-t^2}} \cdot \sqrt{1+4t(1-t^2)}\, dt \longrightarrow k\int t\sqrt{1+4t(1-t^2)}\, dt$$

$$\int \mu' v = \mu v - \int v' \mu$$

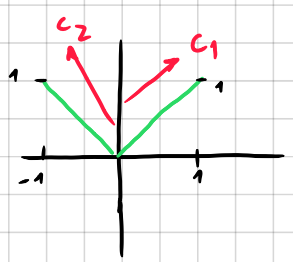
$$\int (\mu \cdot v)' = \int \mu' v + \int v' \mu$$

$$(\mu v)' = \mu' v + v' \mu$$

?

$\ast$ Ejercicio 6 : Hallar masa alambre V en $\mathbb{R}$ $y=|x|$ entre $(-1,1)$ y $(1,1)$, densidad $\delta \overset{k}{=} |xy|$

⇒ Figura de análisis



Masa total = masa alambre $x\geq 0$ + masa alambre $x<0$ $M_1+M_2=\frac{2\sqrt{2}}{3}k$

⇒ para $x\geq 0$ $C_1: \vec{g}_1(t)=(t,t)$

↳ $g'(t)=(1,1) \longrightarrow \|g'(t)\|=\sqrt{2}$

↳ $\delta(g(t))=|t^2|$

$M_1 \overset{k}{=} \int_0^1 t^2 . \sqrt{2}\, dt = \frac{\sqrt{2}\,t^3}{3}\Big|_0^1 = \frac{\sqrt{2}}{3}k$

⇒ para $x<0$ $C_2: \vec{g}_2(t)=(-t,t)$

↳ $g_2'(t)=(-1,1) \longrightarrow g'(t)=\sqrt{2}$

↳ $\delta(g(t))=|-t^2|=t^2$

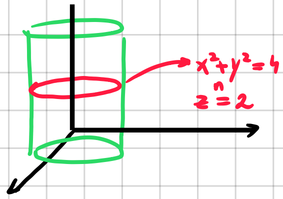
$M_2 \overset{k}{=} \int_{-1}^{0} t^2\sqrt{2}\,dt = \frac{\sqrt{2}\,t^3}{3}\Big|_{-1}^{0} = -\frac{\sqrt{2}\,(-1)^3}{3} = \frac{\sqrt{2}}{3}k$

$\boxed{22/5}$

$\ast$ Ejercicio 7 :

dist $x=0, y=0, z=0$

⇒ forma de alambre



básicamente una circunferencia ↳ $\boxed{z=2}$

⇒ Densidad en cada punto (x,y) o (x,y,z)?

↳ "proporcional al producto de las distancias al plano coord

$\delta(x,y,z) = k\cdot|x||y||z|$

⇒ parametrizar curva $\vec{g}(t)=(2\cos(t), 2\,\mathrm{sen}(t), 2)$, $t\in[0,2\pi]$

↳ $g'(t)=(-2\,\mathrm{sen}(t), 2\cos(t), 0)$ $\|g'(t)\|=\sqrt{(2\cos t)^2+(2\,\mathrm{sen}\,t)^2}=2$

⇒ Hallar $\delta(\vec{g}(t))$

$\delta(\vec{g}(t)) = k\,|2\cos(t)|\,|2\,\mathrm{sen}(t)|\,|2 = 8k\,|\cos(t)\,\mathrm{sen}(t)|$ $M=\int_0^{2\pi} 16\,k\,|\cos(t)\,\mathrm{sen}(t)|\,dt$

$= 8k\int_0^{2\pi} \mathrm{sen}(2t)\,dt = 8k\,\frac{-\cos(2t)}{2} = -4k\cos(2t)\Big|_0^{2\pi} = -4$

$u=2t$
$du=2dt$

$\left(-4k\cos(4\pi)\right) - \left(-4k\cos(0)\right) =$

? wtf

* Ejercicio 8 Hélice : $\vec{x} = \vec{g}(t) = (\cos(t), \text{sen}(t), t), \; t \in [0, 2\pi]$ $\delta(x,y,z) = k(x^2+y^2+z^2)$

Pasos:
- hallar masa total del alambre
- hallar densidad media
- hallar centro de masa → las masas por los dos lados de la curva equivalentes

⇒ Hallar $\|g'(t)\|$

$$g'(t) = (-\text{sen}(t), \cos(t), 1) \Rightarrow \|g'(t)\| = \sqrt{\cos^2+\text{sen}^2+1} = \sqrt{2}$$

⇒ Hallar $\delta(\vec{g}(t))$

$$\delta(\vec{g}(t)) = k\left(\cos(t)^2 + \text{sen}(t)^2 + t^2\right) = k(1+t^2)$$

⇒ Hallar Masa total → $M = \int_0^{2\pi} k(1+t^2)\sqrt{2}\,dt = \sqrt{2}k\int_0^{2\pi}(1+t^2)\,dt = \sqrt{2}k\left(\int_0^{2\pi} 1\,dt + \int_0^{2\pi} t^2\,dt\right)$

$$M = \sqrt{2}k\left(t\Big|_0^{2\pi} - \frac{t^3}{3}\Big|_0^{2\pi}\right) = 12k\left(2\pi - \frac{(2\pi)^3}{3}\right) = 12k\left(\frac{6\pi - (2\pi)^3}{3}\right)$$

$$M = 4k\left(6\pi - (2\pi)^3\right)$$

tengo Down → $\sqrt{2}k\left(2\pi + \frac{(2\pi)^3}{3}\right)$

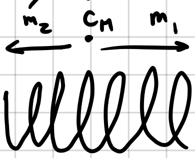
⇒ Hallar longitud alambre

$$\text{longitud alambre} = \int_0^{2\pi}\|g'(t)\|\,dt = \int_0^{2\pi}\sqrt{2}\,dt = \sqrt{2}t\Big|_0^{2\pi} = \sqrt{2}\cdot 2\pi$$

⇒ Hallar densidad media

$$\delta_{media} = M_{total}/\text{Long} = \frac{4k(6\pi-(2\pi)^3)}{\sqrt{2}\cdot 2\pi} = \frac{2k(6-8\pi^2)}{\sqrt{2}}$$

⇒ Hallar centro de masa



Alambre → Acá estarían buscando pto medio ↓ no es lo mismo que centro de masa

→ función $m_1(x) = \int_0^x \delta(g(t))\cdot\|g'(t)\|\,dt = 12\sqrt{k}\left(t\Big|_0^x - \frac{t^3}{3}\Big|_0^x\right)$

$$m_1(x) = 12\sqrt{k}\left(x - \frac{x^3}{3}\right)$$

$$m_2(x) = \int_x^{2\pi}\delta(\vec{g}(t))\cdot\|g'(t)\|\,dt = 12\sqrt{k}\left(t\Big|_x^{2\pi} - \frac{t^3}{3}\Big|_x^{2\pi}\right)$$

$$m_2(x) = 12\sqrt{k}\left(2\pi - x - \left(\frac{(2\pi)^3}{3} - \frac{x^3}{3}\right)\right)$$

Hallar X para que $m_1(x) = m_2(x)$

$$12\sqrt{k}\left(x - \frac{x^3}{3}\right) = 12\sqrt{k}\left(2\pi - x - \frac{(2\pi)^3}{3} + \frac{x^3}{3}\right)$$

$$x - \frac{x^3}{3} = 2\pi - x - \frac{(2\pi)^3}{3} + \frac{x^3}{3}$$

$$2x - \frac{2x^3}{3} = 2\pi - \frac{(2\pi)^3}{3} \rightarrow \boxed{x = \pi}$$

→ punto $\vec{g}(\pi) = (1, 0, \pi)$
centro de masa

(Rehacer ejercicio 8)

* Ejercicio 8 Hélice : $\vec{x} = \vec{g}(t) = (\cos(t), \operatorname{sen}(t), t)$, $t \in [0, 2\pi]$ $\delta(x,y,z) = k(x^2+y^2+z^2)$

Pasos :
- hallar masa total del alambre
- hallar densidad media
- hallar centro de masa → las masas por los dos lados de la curva equivalentes

⇛ Hallar $\|g'(t)\|$

$$g'(t) = (-\operatorname{sen}(t), \cos(t), 1) \Rightarrow \|g'(t)\| = \sqrt{\cos^2+\operatorname{sen}^2+1} = \sqrt{2}$$

⇛ Hallar $\delta(\vec{g}(t))$

$$\delta(\vec{g}(t)) = k\left(\cos(t)^2 + \operatorname{sen}(t)^2 + t^2\right) = k(1+t^2)$$

⇛ Hallar Masa total ⟶ $M = \int_0^{2\pi} k(1+t^2)\sqrt{2}\,dt = \sqrt{2}k\int_0^{2\pi}(1+t^2)\,dt = \sqrt{2}k\left(\int_0^{2\pi} 1\,dt + \int_0^{2\pi} t^2\,dt\right)$

$$M = \sqrt{2}k\left(t\Big|_0^{2\pi} + \frac{t^3}{3}\Big|_0^{2\pi}\right) = \sqrt{2}k\left(2\pi + \frac{(2\pi)^3}{3}\right)$$

$$M = \sqrt{2}k\left(2\pi + \frac{(2\pi)^3}{3}\right)$$

⇛ Hallar longitud alambre

$$\text{longitud alambre} = \int_0^{2\pi}\|g'(t)\|\,dt = \int_0^{2\pi}\sqrt{2}\,dt = \sqrt{2}t\Big|_0^{2\pi} = \sqrt{2}.2\pi$$

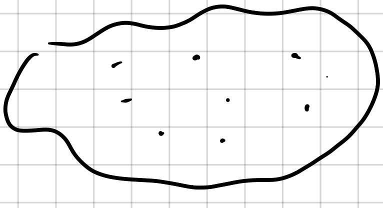
⇛ Hallar densidad media

$$\delta_{media} = M_{total}/\text{Long} = \frac{\sqrt{2}k\left(2\pi + (8\pi^3)/3\right)}{\sqrt{2}\,2\pi} = \underbrace{k\left(1+\frac{4\pi^2}{3}\right)}_{\text{masa media}}$$

⇛ Hallar centro de masa

$\vec{r}(t) = (x(t), y(t), z(t))$ $M = \int_C \delta\,ds$

$\vec{r}_{CM} = \frac{1}{M}\int_C \vec{r}.\delta\,ds$



$$CM = \frac{\sum \text{importancia} \times \text{posicion}}{\text{masa total}}$$

$$\vec{R}_{CM} = \frac{\sum m_1\vec{r}_1 + m_2\vec{r}_2 + m_n\vec{r}_n}{m_{tot}}$$

→ Hallar x_{pos}

$$x_{pos} = \frac{1}{M}\int_0^{2\pi}\cos(t)\,\delta(\vec{g}(t))\,\|g'(t)\|\,dt = \frac{\sqrt{2}k}{M}\int_0^{2\pi}\cos(t)(1+t^2)\,dt = \operatorname{sen}t(1+t^2) - \left(\underset{\downarrow}{\int 2t\operatorname{sen}(t)\,dt}\right)$$

$$(-2t\cos t - 2\int -\cos t\,dt)$$

$u = (1+t^2) \longrightarrow u' = (2t)$ $u = 2t \longrightarrow u' = 2$

$v' = \cos(t) \longrightarrow v = \operatorname{sen}(t)$ $v' = \operatorname{sen}(t) \longrightarrow v = -\cos t$

$$x_{cm} = \frac{\sqrt{2}k}{M} \cdot -2\pi$$

$$= \operatorname{sen}(t)(1+t^2) - (2t\cos t + 2\operatorname{sen}t)\Big|_0^{2\pi}$$

$$\operatorname{sen}(2\pi)(1+(2\pi)^2) - (2\pi\cos(2\pi) + 2\operatorname{sen}(2\pi))$$

que carajo

$$= -2\pi$$

* **Ejercicio 9** Inercia $\int r^2 . dm = \int r^2 \delta\, ds = \int r^2\, \delta(g(t)) \|g'(t)\|\, dt$

↳ alambre $\vec{\sigma}(t) = [-a, a] \subset \mathbb{R} \to \mathbb{R}^2 \,/\, \vec{\sigma}(t) = (t, \cosh(t))$

dist eje $y = |x|$
dist eje $x = |y|$

$I_x = \int \underbrace{|y|^2}_{\cosh^2} \delta(\vec{\sigma}(t)) \|\vec{\sigma}'(t)\|\, dt \qquad I_y = \int \underbrace{|x|^2}_{t} \delta(\vec{\sigma}(t)) \|\vec{\sigma}'(t)\|\, dt$

⇒ Hallar $\|\vec{\sigma}'(t)\|$

$\sigma'(t) = (1, -\operatorname{senh} t) \qquad \|\vec{\sigma}'(t)\| = \sqrt{1 + \operatorname{senh} t} = \sqrt{\cosh^2}$ $\qquad \cosh^2 = \operatorname{senh}^2 + 1$ $\qquad \boxed{\cosh^2 - \operatorname{senh}^2 = 1}$

⇒ Hallar $\delta(\vec{\sigma}(t)) = \delta_0$

$$I_x = \int_{-a}^{a} \delta_0 . (\cosh t)^2 \sqrt{1 + \operatorname{senh} t)^2}\, dt = \delta_0 \int_{-a}^{a} (\cosh t)^2 \sqrt{\cosh t^2}\, dt = \delta_0 \int_{-a}^{a} \cosh t^3\, dt$$

$$\delta_0 \int_{-a}^{a} \cosh t^3\, dt = \delta_0 \int (\operatorname{senh}^2 + 1) \cosh^2\, dt = \delta_0 \int \mu\, d\mu = \delta_0 \frac{\mu^2}{2} \Big|_{a}^{a} = \delta_0 . \frac{(\operatorname{senh}^2 + 1)^2}{2} \Big|_{-a}^{a}$$

$\mu = \operatorname{senh}^2 + 1$
$d\mu = \cosh^2 dt$

$$I_y = \int_{-a}^{a} \delta_0\, t^2 \cosh t\, dt = -t^2 \operatorname{senh} - 2t \cosh + 2 \operatorname{senh} \Big|_{-a}^{a} \qquad ?$$

$$\int \delta_0\, t^2 \cosh t\, dt = -t^2 \operatorname{senh} + \int 2t \operatorname{senh}\, dt = -t^2 \operatorname{senh} + 2t \cdot \cosh + 2\int \cosh dt$$

$\mu = t^2 \to \mu' = 2t \qquad \mu = 2t \to \mu' = 2$
$v' = \cosh \to v = -\operatorname{senh} \qquad v' = \operatorname{senh} \to v = -\cosh$

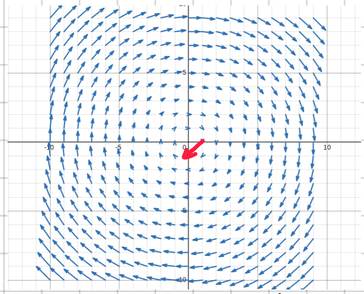
* **Ejercicio 11**

$C:\ \lambda = [a,b] \subset \mathbb{R} \to \mathbb{R}^2 \quad \lambda \in C^1 \quad \operatorname{lon}(C) = 4$
es N_3 de $f = U \subset \mathbb{R}^2 \to \mathbb{R}$ $\Big] \to$ long $C = \int_a^b \|\lambda'\|\, dt = 4$
$f(\lambda) = 3$

$$\int_C f\, ds = \int_a^b f(\lambda) . \|\lambda'\|\, dt = 3 \int_a^b \|\lambda'\| dt = 3 . 4 = 12$$

Valor medio de f es 3 ya que es constante en C
↳ porque C es N_3

Integrales de campos vectoriales (Circulación)

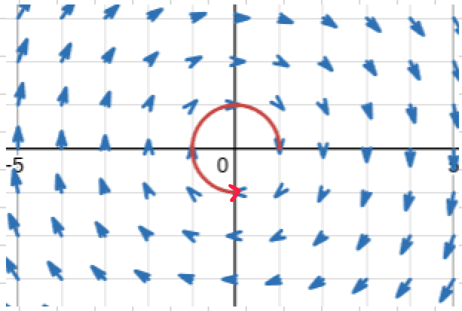


* **Ejercicio 12** : Circulación de $\vec{f}(x,y) = (y,-x)$ $(1,0)$ a $(0,-1)$

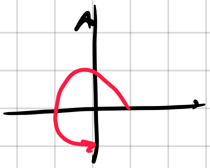
(a) Segmento que los une $\vec{\lambda}(t) = t(1,1)+(1,0) = (t+1,\ t)$

$$\int_C \vec{f}\cdot d\vec{s} = \int_0^{-1} \vec{f}(\vec{\lambda}(t))\cdot \lambda'(t)\ dt = \int_0^{-1} (t,-t-1)(1,1)\,dt = \int_0^{-1} t-t-1\,dt = -t\Big|_0^{-1} = 1$$

$\vec{f}(\vec{\lambda}(t)) = (t,-t-1)$ $\quad \lambda'(t) = (1,1)$



(b) 3/4 circunferencia radio 1

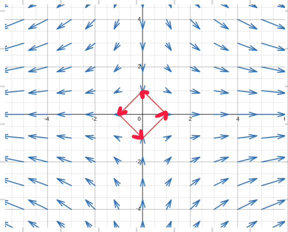


$\vec{g}(t) = (\cos(t), \operatorname{sen}(t)),\ t\in[0, \frac{3\pi}{2}]$

$\vec{f}(\vec{g}(t)) = (\operatorname{sen}(t), -\cos(t))$ $\quad \vec{g}'(t) = (-\operatorname{sen}(t), \cos(t))$

$$\int_C \vec{f}\,d\vec{s} = \int_0^{\frac{3\pi}{2}} (\operatorname{sen}(t), -\cos(t))(-\operatorname{sen}(t), \cos(t))\,dt$$

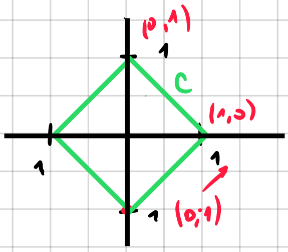
$$= \int_0^{\frac{3\pi}{2}} -\operatorname{sen}^2 t - \cos^2 t\ dt = -\int_0^{\frac{3\pi}{2}} 1\,dt = -t\Big|_0^{\frac{3\pi}{2}} = -\frac{3\pi}{2}$$



* **Ejercicio 13** $\longrightarrow$ $f(x,y) = (2x,-y)$

(a) $\int_{C^+} (2x,-y)\,d\vec{s}$ $\qquad C: |x|+|y| = 1$

$C = C_1 + C_2 + C_3 + C_4$



$$\int_{C^+} \vec{f}\,d\vec{s} = \int_{C_1^+} \vec{f}\,d\vec{s} + \int_{C_2^+} \vec{f}\,d\vec{s} + \int_{C_3^+} \vec{f}\,d\vec{s} + \int_{C_4^+} \vec{f}\,d\vec{s} = \left(2\cdot\frac{3}{2}\right) + 2\cdot\left(\frac{-3}{2}\right) = 0$$

C_1: $\vec{g}_1(t) = (t, t-1)$ $\quad \vec{g}_1'(t) = (1,1)$ $\quad \vec{f}(\vec{g}(t)) = (2t, -t+1)$

$t\in[0,1]$

$$\int_{C_1^+} \vec{f}\,d\vec{s} = \int_0^1 (2t, 1-t)(1,1)\,dt = \int_0^1 2t+1-t = t+\frac{t^2}{2}\Big|_0^1 = 1+\frac{1}{2} = \frac{3}{2}$$

C_2: $\vec{g}_2(t) = (-t+1, t)$ $\quad g'(t) = (-1,1)$ $\quad f(g(t)) = (-2t+2, -t)$

$$\int_{C_2^+} \vec{f}\,ds = \int_0^1 (-2t+2, -t)(-1,1)\,dt = \int_0^1 2t-2-t = \int_0^1 t-2\,dt = \frac{t^2}{2}-2t\Big|_0^1 = \frac{-3}{2}$$

C_3: $\vec{g}_3(t) = (-t, -t+1)$ $\quad \vec{g}'(t) = (-1,-1)$ $\quad \vec{f}(\vec{g}(t)) = (-2t, t-1)$

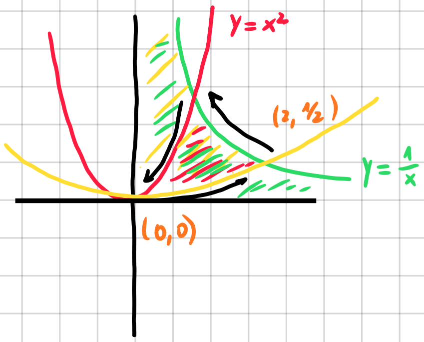
$$\int_{C_3^+} \vec{f}\,d\vec{s} = \int_0^1 (-2t, t-1)(-1,-1)\,dt = \int_0^1 2t-t+1\,dt = \frac{t^2}{2}+t\Big|_0^1 = \frac{1}{2}+1 = \frac{3}{2}$$

C_4: $\vec{g}_4(t) = (t-1, -t)$ $\quad \vec{g}'(t) = (1,-1)$ $\quad \vec{f}(\vec{g}(t)) = (2t-2, t)$

$$\int_{C_4^+} \vec{f}\,d\vec{s} = \int_0^1 (2t-2, t)(1,-1)\,dt = \int_0^1 2t-2-t = \frac{t^2}{2}-2t\Big|_0^1 = \frac{1}{2}-2 = -\frac{3}{2}$$

ⓑ $\int_{C^+} (xy, x^2)\, d\vec{s}$ C es frontera región primer cuadr $xy \le 1$, $y \le x^2$, $8y \ge x^2$

$\Rightarrow$ Hallar intersección $xy \le 1$, $y \le x^2$, $8y \ge x^2$ $f(x,y) = (xy, x^2)$



Hallar punto de intersección

$y = \frac{1}{8}x^2$ $y = \frac{1}{x}$

$\frac{1}{x} = \frac{1}{8}x^2 \to x^3 = 8 \to \boxed{x=2}$

punto $(2, \frac{1}{2})$

$y = \frac{1}{x}$ $y = x^2$

$x^2 = \frac{1}{x} \Rightarrow x^3 = 1$, $\boxed{x=1}$

punto $(1,1)$

$$\boxed{C = C_1 + C_2 + C_3}$$

C_1: $\vec{g}_1(t) = (t, \frac{1}{8}t^2)$, $t \in [0,2]$ $\vec{g}_1'(t) = (1, \frac{1}{4}t)$ $f(g(t)) = (\frac{1}{8}t^3, t^2)$

C_2: $\vec{g}_2(t) = (t, 1/t)$, $t \in [2,1]$ $\vec{g}_2'(t) = (1, -1/t^2)$ $f(g_2(t)) = (1, t^2)$

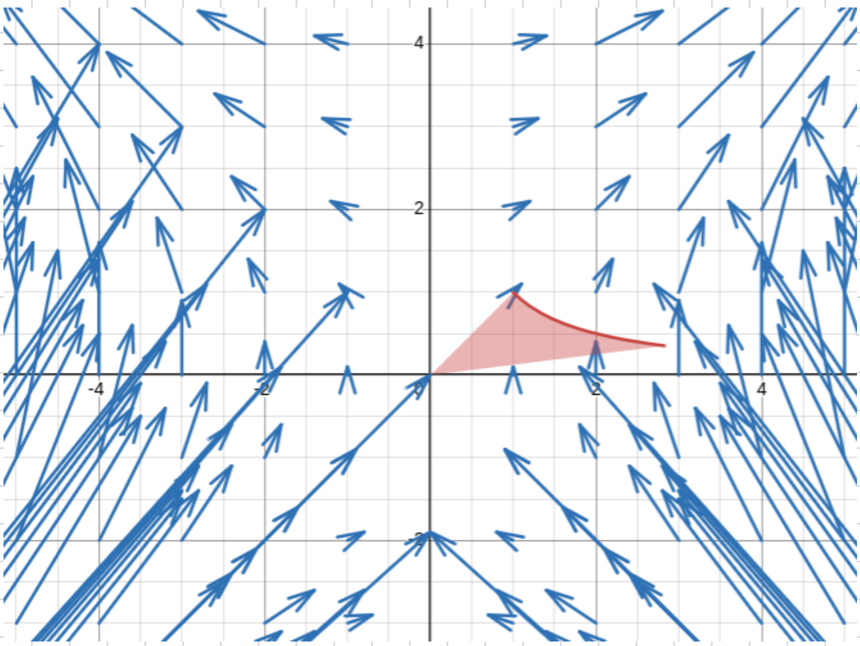
C_3: $\vec{g}_3(t) = (t, t^2)$, $t \in [1,0]$ $\vec{g}_3'(t) = (1, 2t)$ $f(g_3(t)) = (t^3, t^2)$

$$\int_{C_1^+} \vec{f}\, d\vec{s} = \int_0^2 (\tfrac{1}{8}t^3, t^2)(1, \tfrac{1}{4}t)\, dt = \int_0^2 \tfrac{1}{8}t^3 + \tfrac{1}{4}t^3\, dt = \frac{3}{8}\,\frac{t^4}{4}\Big|_0^2 = \frac{3}{2}$$

$$\int_{C_2^+} \vec{f}\, d\vec{s} = \int_2^1 (1, t^2)(1, -1/t^2)\, dt = \int_2^1 1 - 1\, dt = \int_2^1 0\, dt = 0$$

$$\int_{C_3^+} \vec{f}\, d\vec{s} = \int_1^0 (t^3, t^2)(1, 2t)\, dt = \int_1^0 t^3 + 2t^3\, dt = \frac{3t^4}{4}\Big|_1^0 = -\frac{3}{4}$$

$$\int_{C^+} \vec{f}\, d\vec{s} = \int_{C_1^+} \vec{f}\, d\vec{s} + \int_{C_2^+} \vec{f}\, ds + \int_{C_3^+} \vec{f}\, d\vec{s} = \frac{3}{2} + 0 + \frac{-3}{4} = \frac{3}{4}$$



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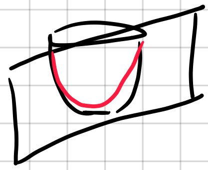
* Ejercicio 14

(a) $\underbrace{\vec{\sigma}(t) = (t, t^2, 2t)}_{C}$ $\qquad \vec{f}(x,y,z) = (xy, x, zy)$



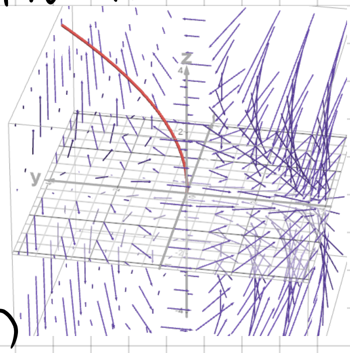
→ C es una parábola → probablemente intersección de paraboloide con un plano

⇒ Hallar datos necesarios

$$\vec{f}(\vec{\sigma}(t)) = (t^3, t, 2t^3) \qquad \vec{\sigma}'(t) = (1, 2t, 2)$$

$$\int_C \vec{f}\, d\vec{s} = \int_0^2 \vec{f}(\vec{\sigma}(t))\, \vec{\sigma}'(t)\, dt = \int_0^2 (t^3, t, 2t^3)(1, 2t, 2)\, dt = \int_0^2 t^3 + 2t^2 + 4t^3\, dt$$

$$= \int_0^2 5t^3 + 2t^2\, dt = \frac{5}{4}t^4 + \frac{2}{3}t^3 \Big|_0^2 = \frac{76}{3}$$



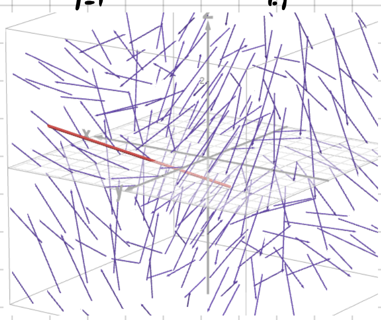
punto inicial es $\vec{\sigma}(0) = (0,0,0)$ y $\vec{\sigma}(2) = (2,4,4)$

(b) $\vec{\sigma}(t) = (t+1, 2t+1, t) \quad t \in [-1, 2] \qquad \vec{f}(x,y,z) = (x+2y+z, 2y, 3x-z)$

$$\vec{f}(x,y,z) = ((t+1)+2(2t+1)+t,\ 2(2t+1),\ 3(t+1)-t) \qquad \vec{\sigma}'(t) = (1, 2, 1)$$

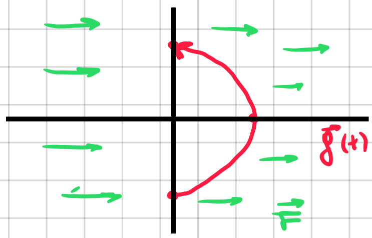
$$\int_C \vec{f}\, d\vec{s} = \int_{-1}^2 (t+1+4t+2+t,\ 4t+2,\ 3t+3-t)(1,2,1)\, dt = \int_{-1}^2 (6t+3, 4t+2, 2t+3)(1,2,1)\, dt$$

$$\int_{-1}^2 6t+3+8t+4+2t+3\, dt = \int_{-1}^2 16t+10\, dt = \frac{16}{2}t^2 + 10t \Big|_{-1}^2 = 8t^2 + 10t \Big|_{-1}^2 = 52 - (-2) = 54$$



* Ejercicio 15 $\quad \vec{F}(x,y,z) = 2\cdot \hat{j} = (0,2)$

⇒ f.g análisis curva + cam vect $\qquad W_F$



⇒ parametrizar circunferencia $(0,-1) \to (1,0) \to (0,1)$

$$\vec{g}(t) = (\text{sen}(t), -\cos(t)), \quad t \in [0, \pi]$$

$$\vec{g}(0) = (0,-1), \quad \vec{g}\left(\tfrac{\pi}{2}\right) = (1, 0), \quad \vec{g}(\pi) = (0, 1)$$

$$\vec{F}(\vec{g}(t)) = (0,2) \qquad \vec{g}'(t) = (\cos(t), \text{sen}(t)) \quad t \in [0, \pi]$$

$$W_F = \int_0^\pi (0,2)(\cos t, \text{sen}\, t)\, dt = 2\int_0^\pi \text{sen}\, t\, dt = -2\cos t \Big|_0^\pi = (\underbrace{-2\cos(\pi)}_{2}) - (\underbrace{-2\cos(0)}_{-2}) = 4$$

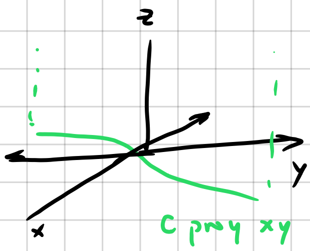
* Ejercicio 1b

$\vec{F}(x,y,z) = (2g(x,y,z),\ xy - 9xg(x,y,z),\ 3yg(x,y,z))$, $g = \mathbb{R}^3 \to \mathbb{R}$

↪ Calc circulación de $\vec{F}$ desde $(1, y_0, z_0)$ a $(8, y_1, z_1)$ $\quad C \in \{z = x - y^2\}$ y C proy xy, $x = y^3$

→ Hallar C, parametrizar C, hallar puntos inicial/final, calcular circulación

⇒ Figura de análisis C

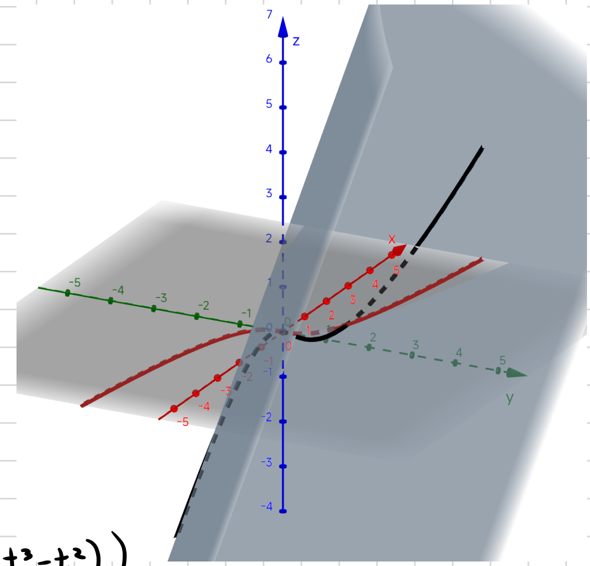


básicamente la proyección al plano xy quiere decir que independiente de z. $\boxed{x = y^3}$

→ $C \in \{(x,y,z) / x = y^3\}$

Info $\quad C = \begin{cases} y = y \\ x = y^3 \\ z = x - y^2 = y^3 - y^2 \end{cases}$

$\vec{\sigma}(t) = (t^3, t, t^3 - t^2)$



⇒ Hallar valor inicial y final de t → $t \in [1, 2]$

↳ hallar $t_0 \,/\, \vec{\sigma}(t_0) = (1, y_0, z_0)$

$$\begin{cases} t_0^3 = 1 \rightarrow \boxed{t_0 = 1} \\ t_0 = y_0 \\ t_0^3 - t_0^2 = z_0 \end{cases}$$

↳ Hallar $t_1 \,/\, \vec{\sigma}(t_1) = (8, y_1, z_1)$

$$\begin{cases} t_1^3 = 8 \rightarrow \boxed{t_1 = 2} \\ t_1 = y_1 \\ t_1^3 - t_1^2 = z_1 \end{cases}$$

⇒ Hallar datos necesarios para calcular circulación

$$\vec{F}(\vec{\sigma}(t)) = \left(2.\vec{g}(t^3, t, t^3 - t^2),\ t^4 - 9t^3\vec{g}(t^3, t, t^3 - t^2),\ 3t\vec{g}(t^3, t, t^3 - t^2)\right)$$

$$\vec{\sigma}'(t) = (3t^2, 1, 3t^2 - 2t)$$

⇒ Calcular circulación

resumir $\vec{g}(t^3, t, t^3 - t^2) = \vec{g}$

$$\int_1^2 \vec{F}(\vec{\sigma}(t)) \cdot \vec{\sigma}'(t)\, dt = \int_1^2 \left(2.\vec{g}(t^3,t,t^3-t^2),\ t^4 - 9t^3\vec{g}(t^3,t,t^3-t^2),\ 3t\vec{g}(t^3,t,t^3-t^2)\right)(3t^2, 1, 3t^2 - t^2)\, dt$$

$$= \int_1^2 (2\vec{g},\ t^4 - 9t^3\vec{g},\ 3t\vec{g})(3t^2, 1, 3t^2 - 2t)\, dt = \int_1^2 6t^2\vec{g} + t^4 - 9t^3\vec{g} + 9t^3\vec{g} - 6t^2\vec{g}\, dt$$

$$= \int_1^2 t^4\, dt = \frac{t^5}{5}\Big|_1^2 = \frac{32}{5} - \frac{1}{5} = \frac{31}{5}$$

↳ El resultado no depende de $\vec{g}$

* Ejercicio 17

$\vec{f}(x,y,z) = (x+\vec{g}(xy+z),\ y+\vec{g}(xy+z),\ 2z)$ $\qquad g = \mathbb{R}^3 \to \mathbb{R} / \vec{g}(x,y,z) = xy+z$

$A=(2,0,3)$ $\qquad B=(3,-1,6)$ $\qquad$ mostrar $\vec{F}_{AB}$ circulación no dep. $\vec{g}$

→ Hallar curva que pasa por seg $\overline{AB}$

$\vec{AB} = B-A = (1,-1,3)$ $\qquad \vec{\sigma}(t) = t(1,-1,3)+(2,0,3) = (t+2,-t,3t+3)$, $t \in [0,1]$

→ Hallar datos para calcular circulación

$\vec{f}(\vec{\sigma}(t)) = (t+2+\vec{g},\ -t+\vec{g},\ 6t+6)$ $\qquad \vec{\sigma}\,'(t) = (1,-1,3)$

→ calcular circulación

$$\int_0^1 (t+2+\vec{g}, -t+\vec{g}, 6t+6)(1,-1,3)\,dt = \int_0^1 t+2+\vec{g}+t-\vec{g}+18t+18\,dt$$

$$= \int_0^1 20t+20\,dt = 10t^2+20t\Big|_0^1 = 30$$

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Campos de Gradientes

* Ejercicio 18 : Analizar si campo admite función potencial $\Rightarrow$ Hallar $\exists \phi / \vec{f} = \nabla \phi$ condición necesaria

ⓐ $\vec{f}(x,y) = (2x + y^2 sen(2x),\ 2y sen^2(x))$ Dom f = $(x,y) \in \mathbb{R}^2$

$$Df = \begin{pmatrix} 2+2y^2\cos(2x) & 2y\,sen(2x) \\ \underbrace{4y\,sen(x)\cos(x)}_{2y\,sen(2x)} & 2sen^2(x) \end{pmatrix}$$

$\rightarrow \vec{f}$ cumple con condición necesaria Df simétrica

$\rightarrow$ Dominio Dom $\vec{f}$ abierto y simp conexo

↳ Hallar función potencial $\phi(x,y)$

$\frac{\partial \phi}{\partial y} = Q(x,y) = 2y\,sen^2 x$

$\phi(x,y) = \int Q(x,y)\,dy = 2sen^2 x \cdot \frac{y^2}{2} + c(x)$

función potencial $\phi(x,y) = y^2 sen^2(x) + x^2 + k$

↳ hallar $c(x)$

$c'(x) = 2x \rightarrow c(x) = x^2$

$\frac{d}{dx}\left(\int Q(x,y)\,dy\right) = 2sen x . \cos x\ y^2 + c'(x) \Rightarrow 2sen x \cos x\ y^2 + c'(x) = 2x + y^2 sen(2x)$

ⓑ $\vec{f}(x,y,z) = (xy,\ x+zy,\ yz)$ $P = xy$ $Q = x+zy$ $R = yz$

↳ Ver si: $D\vec{f}$ simétrica

$$D_f = \begin{pmatrix} P_x' & P_y' & P_z' \\ Q_x' & Q_y' & Q_z' \\ R_x' & R_y' & R_z' \end{pmatrix} \quad \nabla \times \vec{f} = \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{pmatrix} = (R_y' - Q_z',\ P_z' - R_x',\ Q_x' - P_y') = (0,0,0)$$

↳ $P_y' = x \Rightarrow Q_x' = 1$ (X) $P_y' \neq Q_x' \rightarrow D\vec{f}$ no es simétrica

---

ⓒ $\vec{f}(x,y,z) = (y - 2xz + 1,\ x + 2y,\ -x^2)$ $P = y - 2xz + 1$, $Q = x + 2y$. $R = -x^2$

$\Rightarrow$ Ver matriz sim

$$D_f = \begin{pmatrix} P_x' & P_y' & P_z' \\ Q_x' & Q_y' & Q_z' \\ R_x' & R_y' & R_z' \end{pmatrix}$$

$\rightarrow P_y' = 1$, $Q_x' = 1$

$\rightarrow P_z' = -2x$, $R_x' = -2x$

$\rightarrow Q_z' = 0$, $R_y' = 0$

$D\vec{f}$ es simétrica y Dom f abierto y simpl conexo

$\exists \phi / \nabla\phi = f$

$\Rightarrow$ Hallar ϕ

$\phi = \int R(x,y,z) \cdot dz = \int -x^2\,dz = \underline{-x^2 z + \alpha_1(x) + \alpha_2(y)}$ $\frac{\partial}{\partial z}$

↳ hallar $\alpha_1(x)$ y $\alpha_2(y)$

$\phi(x,y,z) = -x^2 z + yx + x + y^2 + k$, $k \in \mathbb{R}$

$\frac{\partial}{\partial x}(-x^2 z + \alpha_1(x) + \alpha_2(y)) = -2xz + \alpha_1'(x) \Rightarrow -2xz + \alpha_1'(x) = y - 2xz + 1 \Rightarrow \alpha_1'(x) = y + 1 \rightarrow \alpha_1(x) = yx + x$

$\frac{\partial}{\partial y}(-x^2 z + yx + x + \alpha_2(y)) = x + \alpha_2(y) \Rightarrow \alpha_2'(y) = 2y \rightarrow \alpha_2(y) = y^2$

(d) $\vec{F}(x,y,z) = \left((1+xz)e^{xz},\ xe^{xz},\ yx^2e^{xz} \right)$, $\text{Dom } f = \mathbb{R}^3$

$\Rightarrow$ ver matriz sim

$\rightarrow P'_y = 0$. $Q'_x = 1e^{xz} + ze^{x}x$

$$D_f = \begin{pmatrix} P'_x & P'_y & P'_z \\ Q'_x & Q'_y & Q'_z \\ R'_x & R'_y & R'_z \end{pmatrix} \rightarrow P'_z = xe^{xz} + xe^{xz}(1+xz),\quad R'_x = 2xye^{xz} + ze^{xz}x^2y$$

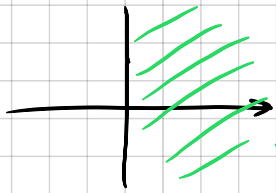
$\rightarrow$ no admite potencial

no es simétrica Df

---

* Ejercicio 19

$\vec{f}(x,y) = \left(y + xg(x),\ 3y + xg(x) \right)$, $\vec{f}(1,1) = (3,5)$, hallar $g(x)$ para func pot $\vec{f}$ semiplano $x>0$



$f(x,y) / x>0$

$\Rightarrow$ Ver valor $g(1)$

$\vec{f}(1,1) = (1 + g(1),\ 3 + g(1)) = (3,5) \longrightarrow \boxed{g(1) = 2}$

$\underbrace{g(1)}_{2} + g'(1) = 1 \Rightarrow \boxed{g'(1) = -1}$

$\Rightarrow$ Hallar Df para que sea simétrica

$$D_f = \begin{pmatrix} g(x) + g'(x)x & 1 \\ g(x) + xg'(x) & 3 \end{pmatrix} \Rightarrow \boxed{\begin{array}{l} g(x) + xg'(x) = 1 \\ \forall x \in \mathbb{R} \end{array}}$$ para que sea simétrico

$\downarrow$

podría ser una recta

$\boxed{\begin{array}{l}\text{recta} \\ g(x) = -x + 3\end{array}}$

$g(1) = g'(1) \cdot 1 + b = 2$

$\boxed{b = 3}$

$g(x) - 1 = -xg'(x)$

$g'(x) = \dfrac{1 - g(x)}{x}$

$dg(x) = \dfrac{1 - g(x)}{x}\, dx$

$$\int \frac{1}{1 - g(x)}\, dg = \int \frac{1}{x}\, dx$$

$\ln|1 - g(x)| = \ln|x| + k$

$|1 - g(x)| = e^{\ln|x|} \cdot \underbrace{e^{k}}_{>0}$

$1 - g(x) = \pm e^{k} e^{\ln|x|} \rightarrow c$

$g(x) = 1 + c \cdot |x| \rightarrow c \in \mathbb{R}$

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* Ejercicio 21

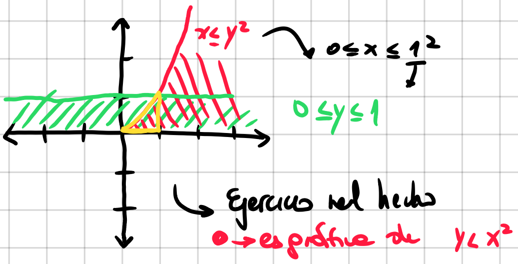
$f(x,y) = (x, x-y^2) \rightarrow P = x, \quad Q = x - y^2, \quad \text{Dom} f = \mathbb{R}^2$

(a) Demostrar f no tiene potencial

$$Df = \begin{pmatrix} 1 & 0 \\ 1 & -2y \end{pmatrix} \longrightarrow$$ como $\vec{f}$ no es simétrica, no tiene potencial

(b) Hallar circulación de f en sent positivo para frontera de región $0 \le y \le 1$, $0 \le x \le y^2$

$\Rightarrow$ graficar región



frontera de la región

$(0,0) \longrightarrow (1,0)$ segmento $\rho_1(t) = (t,0), \; t \in [0,1]$ $\quad C_1$

$(1,0) \longrightarrow (1,1)$ segmento $\rho_2(t) = (1,t), \; t \in [0,1]$ $\quad C_2$

$(1,1) \longrightarrow (0,0)$ parábola (parte) $\rho_3(t) = (-t, t^2) \; t \in [-1, 0]$ $\quad C_3$

cambiar sentido

$\Rightarrow$ Hallar datos necesarios para calcular circulación

$C_1 = \vec{\rho}_1'(t) = (1,0) \qquad \vec{f}(\vec{\rho}_1(t)) = (t,t)$

$$\int_{C_1^+} \vec{f}\, d\vec{s} = \int_0^1 (t,t)(1,0)\, dt = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}$$

$C_2 = \vec{\rho}_2'(t) = (0,1) \qquad \vec{f}(\vec{\rho}_2(t)) = (1, 1-t^2)$

$$\int_{C_2^+} \vec{f}\, d\vec{s} = \int_0^1 (1, 1-t^2)(0,1)\, dt = \int_0^1 1 - t^2\, dt = t - \frac{t^3}{3}\Big|_0^1 = \frac{2}{3}$$

$C_3 = \vec{\rho}_3'(t) = (-1, 2t) \qquad \vec{f}(\vec{\rho}_3(t)) = (-t, -t-t^4)$

$$\int_{C_3^+} \vec{f}\, d\vec{s} = \int_{-1}^0 (-t, -t-t^4)(-1, 2t)\, dt = \int_{-1}^0 t - 2t^2 - 2t^5\, dt$$

$$= \frac{t^2}{2} - \frac{2t^3}{3} - \frac{t^6}{3}\Big|_{-1}^0 = -\left(\frac{1}{2} - \frac{2}{3} - \frac{1}{3}\right) = \frac{1}{2}$$

$$\int_{\partial D^+} \vec{f}\, d\vec{s} = \int_{C_1^+} \vec{f}\, d\vec{s} + \int_{C_2^+} \vec{f}\, d\vec{s} + \int_{C_3^+} \vec{f}\, d\vec{s} = \frac{1}{2} + \frac{2}{3} + \frac{1}{2} = \frac{5}{3}$$

---

* Ejercicio 23

(a) $$\int_C (f \nabla g + g \nabla f)\, d\vec{s} = \int_A^B (f \nabla g + g \nabla f)\, d\vec{s} = f(\vec{x})\, g(\vec{x})\Big|_A^B = f(B)g(B) - f(A)g(A)$$

(b) $$\int_C (2fg \nabla f + f^2 \nabla g)\, d\vec{s}$$

$$\int (2fg \nabla f + f^2 \nabla g)\, d\vec{s}$$

$$\underbrace{2\int fg \nabla f\, d\vec{s}}_{\substack{\mu = f \\ d\mu = f\, ds}} + \underbrace{\int f^2 \nabla g\, ds}_{\substack{\mu = f^2 \rightarrow \mu' = 2f \\ v' = \nabla g \rightarrow v = g}} = 2\int g \cdot \mu\, d\mu + \left(f^2 g - \int 2fg\, ds\right) = 2g\frac{f^2}{2} + f^2 g -$$

$$\int g \cdot \mu\, d\mu$$

(c) $$\int_C \frac{g \nabla f - f \nabla g}{g^2}\, ds = \frac{f}{g}\Big|_A^B = \frac{f(B)}{g(B)} - \frac{f(A)}{g(A)}$$

* Ejercicio 22 $\vec{f}(x,y,z) = \left(4x/z,\ 2y/z,\ -(2x^2+y^2)/z^2\right)$ con $z \neq 0$, $\text{Dom}\,\vec{f} = \mathbb{R}^3 - \{(x,y,z)/z=0\}$

↳ $P = 4x/z$, $Q = 2y/z$, $R = -(2x^2+y^2)/z^2$

(a) demostrar f admite potencial $z>0$

⇒ demostrar matriz simétrica $z>0$

$$D_f = \begin{pmatrix} P_x' & P_y' & P_z' \\ Q_x' & Q_y' & Q_z' \\ R_x' & R_y' & R_z' \end{pmatrix} \qquad P_y' = 0, \quad Q_x' = 0 \checkmark$$

$P_z' = -4x/z^2$, $R_x' = -\frac{1}{z^2} \cdot 4x \checkmark$

$Q_z' = -2y/z^2$ $R_y' = -2y/z^2 \checkmark$

matriz simétrica para $z>0$
y región $z>0$ convexo
entonces $\vec{f}$ admite potencial

⇒ Hallar potencial $\phi(x,y,z)$ para que $\phi(1,1,1) = 3$

$$\phi = \int P(x,y,z)\,dx = \int \frac{4x}{z}\,dx = \frac{1}{z} \cdot \frac{4x^2}{2} + \alpha_1(y) + \alpha_2(z) = \frac{2x^2}{z} + \alpha_1(y) + \alpha_2(z)$$

$$\frac{2x^2}{z} + \frac{y^2}{z} + \alpha_2(z)$$

$\phi(x,y,z) = \dfrac{2x^2+y^2}{z} + \underset{0}{k}$

$\phi(1,1,1) = \dfrac{2 \cdot 1^2 + 1^2}{1^2} = 3$

⇒ Hallar $\alpha_1(y)$ ⇒ $\frac{\partial}{\partial y}(\phi) = Q(x,y,z)$

$$\frac{\partial}{\partial y}\left(\frac{2x^2}{z} + \alpha_1(y) + \alpha_2(z)\right) = \alpha_1'(y) \Rightarrow \alpha_1'(y) = \frac{2y}{z} \Rightarrow \int \alpha_1'(y)\,dy = \int \frac{2y}{z}\,dy \Rightarrow \boxed{\alpha_1(y) = \frac{y^2}{z}}$$

⇒ Hallar $\alpha_2(z)$ $\frac{\partial}{\partial z}(\phi) = R(x,y,z)$

$\boxed{\alpha_2(z) = 0}$

$$\frac{\partial}{\partial z}\left(\frac{2x^2+y^2}{z} + \alpha_2(z)\right) = -\frac{(2x^2+y^2)}{z^2} + \alpha_2'(z) \Rightarrow \alpha_2'(z) - \frac{(2x^2+y^2)}{z^2} = -\frac{(2x^2+y^2)}{z^2} \rightarrow \boxed{\alpha_2'(z) = 0}$$

(b) Hallar circulación de $\vec{f}$ en curva. $x = 1 + \log(1 + |\text{sen}(t)|)$ $y = e^{t(\pi - t)}$ $z = 1 + t/\pi$, $t \in [0, \pi]$

↳ la idea acá no es usar $\int_a^b \vec{f}(\vec{g}(t))\,\vec{g}'(t)\,dt$ sino $\int_C \vec{f}\,d\vec{s} = \phi(B) - \phi(A)$ $B = \vec{g}(b)$ $A = \vec{g}(a)$

$$\Rightarrow \int_C \vec{f}\,ds = \int_a^b \vec{f}(\vec{g}(t))\,\vec{g}'(t)\,dt = \phi(B) - \phi(A)$$

$$B = \vec{g}(\pi) = \left(1 + \log(1 + \text{sen}(\pi)),\ e^{\pi(\pi-\pi)},\ 1 + \pi/\pi\right) = (1, 1, 2)$$

$$A = \vec{g}(0) = \left(1 + \log(1 + \text{sen}(0)),\ e^{0(\pi-0)},\ 1 + 0/\pi\right) = (1, 1, 1)$$

$$\phi(B) = \phi(1,1,2) = \frac{2 \cdot 1^2 + 1^2}{2} = \frac{3}{2}$$

$$\phi(B) - \phi(A) = \frac{3}{2} - 3 = \frac{-3}{2}$$

$$\phi(A) = \phi(1,1,1) = \frac{2 \cdot 1^2 + 1^2}{1} = 3$$

$\boxed{29/5}$

* Ejercicio 24 Calcular trabajo que realiza $\vec{F}$ sobre trayectoria

↳ $\vec{F}(x,y,z) = (y^2\cos(x)+z^3)\,i + (2y\,\text{sen}\,x - 4)\,j + (3xz^2+2)\,k$

$P = y^2\cos x + z^3$
$Q = 2y\,\text{sen}\,x - 4$
$R = 3xz^2 + 2$

↳ $g: \begin{cases} x = \text{arcsen}(t) \\ y = 1-2t \\ z = 3t-1 \end{cases} \quad , 0 \le t \le 1$

como Dom f es conexo   Dom $f = \mathbb{R}^3$
y D_f es simétrica,
$\vec{F}$ campo de gradientes y admite potencial

trabajo $\vec{F}$

$$W_f = \int_c \vec{f}\,d\vec{s} = \int_0^1 \vec{f}(g(t))\cdot \vec{g}'(t)\,dt = \phi(g(t))\Big|_0^1 = \underbrace{\phi(g(1))}_{4\pi+9} - \underbrace{\phi(g(0))}_{-6} = 4\pi+15 \cong 27{,}57$$

Ver si $\vec{F}$ admite potencial → D_f matriz sim

$$Df = \begin{pmatrix} P'_x & P'_y & P'_z \\ Q'_x & Q'_y & Q'_z \\ R'_x & R'_y & R'_z \end{pmatrix} \Rightarrow \begin{array}{ll} P'_y = 2y\cos(x) & Q'_x = 2y\cos(x) \\ P'_z = 3z^2 & R'_x = 3z^2 \\ Q'_z = 0 & R'_y = 0 \end{array}$$

D_f matriz simétrica

⇒ Hallar función potencial ϕ / $\vec{F}(x,y,z) = \nabla\phi$

$$\frac{\partial\phi}{\partial z} = R \Rightarrow \phi = \int R\,dz = \int 3xz^2+2\,dz = 3x\int z^2 dz + \int 2\,dz = 3x\frac{z^3}{3} + 2z + \alpha_1(x) + \alpha_2(y)$$

o Hallar $\alpha_1(x)$ y $\alpha(y)$

$$\frac{\partial\phi}{\partial x} = P \Rightarrow \frac{\partial}{\partial x}\left(xz^3 + 2z + \alpha_1(x) + \alpha_2(y)\right) = \underbrace{z^3 + \alpha_1'(x)}_{\frac{\partial}{\partial x}\phi} = \underbrace{z^3 + y^2\cos(x)}_{P}$$

$$\alpha_1'(x) = y^2\cos(x) \longrightarrow \alpha_1(x) = y^2\int\cos x\,dx = y^2\,\text{sen}\,x$$

$$\frac{\partial\phi}{\partial y} = Q \Rightarrow \frac{\partial}{\partial y}\left(xz^3 + 2z + y^2\,\text{sen}\,x + \alpha_2(y)\right) = 2y\,\text{sen}\,x + \alpha_2'(y) = 2y\,\text{sen}\,x - 4$$

$$\alpha_2'(y) = -4 \Rightarrow \alpha_2(y) = -4y$$

$$\phi(x,y,z) = xz^3 + 2z + y\,\text{sen}\,x + y^2\,\text{sen}\,x - 4y - y\,\text{sen}\,x$$

$$= xz^3 + 2z + y^2\,\text{sen}\,x - 4y + C$$

⇒ Hallar $g(0)$ y $g(1)$

$g(1) = \left(\frac{\pi}{2}, -1, 2\right)$

$g(0) = (0, 1, -1)$

Hallar $\phi(g(1))$ y $\phi(g(0))$

$\phi(g(1)) = \phi\left(\frac{\pi}{2}, -1, 2\right) = \frac{\pi}{2}\cdot 2^3 + 2\cdot 2 + 1 + 4 = 9 + 4\pi$

$\phi(g(0)) = \phi(0,1,-1) = -6$

* Ejercicio 28 : Hallar a y b para $\vec{f}$ conservativo $\boxed{30/5}$

↳ $\vec{f}(x,y,z) = (axsen(\pi y),\ x^2\cos(\pi y) + bye^{-z},\ y^2e^{-z})$

↳ Para que $\vec{f}$ sea conservativo, entonces Dom f convexo y Df simétrico

⇒ Hallar Df simétrico

$$D_f = \begin{pmatrix} P_x' & P_y' & P_z' \\ Q_x' & Q_y' & Q_z' \\ R_x' & R_y' & R_z' \end{pmatrix} \begin{matrix} \Rightarrow P_y' = ax\cos(\pi y).\pi \ , \ Q_x' = 2x\cos(\pi y) \\ \Rightarrow P_z' = 0 \ , \ R_x' = 0 \\ \Rightarrow Q_z' = -bye^{-z} \ , \ R_y' = 2ye^{-z} \end{matrix}$$

Para que D_f sim

$P_y' = Q_x'$ y $Q_z' = R_y'$

$$\begin{cases} ax\cos(\pi y).\pi = 2x\cos(\pi y) & \boxed{a = \frac{2}{\pi}} \\ -bye^{-z} = 2ye^{-z} \rightarrow & \boxed{b = -2} \end{cases}$$

⇒ Hallar circulación

$$\int_C \vec{f}\, d\vec{s} = \int_0^{\pi} \vec{f}(\vec{g}(t))\, g'(t)\, dt = \phi(\vec{g}(\pi)) - \phi(\vec{g}(0)) = C - C = \underline{0}$$

⇒ Hallar ϕ

$$\phi = \int y^2 e^{-z}\, dz = -y^2e^{-z} + \alpha_1(x) + \alpha_2(y)$$

⇒ Hallar $\alpha_1(x)$ y $\alpha_2(y)$

$$\frac{\partial\phi}{\partial x} = P \rightarrow \frac{\partial}{\partial x}\left(-y^2e^{-z} + \alpha_1(x) + \alpha_2(y)\right) = \alpha_1'(x) = \frac{2}{\pi}x\,sen(\pi y) \Rightarrow \alpha_1(x) = \frac{x^2}{\pi}sen(\pi y)$$

$$\frac{\partial\phi}{\partial y} = Q \Rightarrow \frac{\partial}{\partial y}\left(-y^2e^{-z} + \frac{x^2}{\pi}sen(\pi y) + \alpha_2(y)\right) \rightarrow -2ye^{-z} + \frac{x^2}{\pi}\cos(\pi y).\pi + \alpha_2'(y) = x^2\cos(\pi y) - 2ye^{-z}$$

$$\alpha_2'(y) = 0 \rightarrow \alpha_2(y) = 0$$

$$\phi(x,y,z) = -y^2e^{-z} + \frac{x^2}{\pi}sen(\pi y) + C \ , \ C \in \mathbb{R}$$

Hallar $\vec{g}(0)$ y $\vec{g}(\pi)$, $\vec{g}(t) = (\cos(t),\ sen(2t),\ sen^2(t))$

→ $\vec{g}(\pi) = (-1, 0, 0)$

↳ $\vec{g}(0) = (1, 0, 0)$

Hallar $\phi(\vec{g}(\pi))$ y $\phi(\vec{g}(0))$

↳ $\phi(\vec{g}(\pi)) = \phi(-1,0,0) = 0 + \frac{1}{\pi}sen(\pi.0) + C$

↳ $\phi(\vec{g}(0)) = \phi(1,0,0) = 0 + \frac{1}{\pi}sen(\pi.0) + C$

* Ejercicio 29 : $f(x,y,z) = (ax\ln(z))\check{i} + (by^2z)\check{j} + (x^2/z + y^3)\check{k}$

↳ Hallar a,b. y Domf para f conservativo y $\exists f(1,1,1)$

↳ Hallar segmento, limitar segmento

↳ Calcular circulación f en segmento

⇒ Hallar Dom f

$(1,1,1) \in \text{Dom} f$

↳ el dominio de f $\text{Dom} f = \{(x,y,z) \,/\, z>0\}$ ya que hay $\ln(z)$ y $\frac{1}{z}$

⇒ Hallar a,b → Df simétrico

$$D_f = \begin{pmatrix} P'_x & P'_y & P'_z \\ Q'_x & Q'_y & Q'_z \\ R'_x & R'_y & R'_z \end{pmatrix} \quad \begin{aligned} &\to P'_y = 0 && Q'_x = 0 \\ &\to P'_z = ax\frac{1}{z} && R'_x = \frac{2x}{z} \\ &\to Q'_z = by^2 && R'_y = 3y^2 \end{aligned}$$

Para que f sea conservativa entonces $a=2$ y $b=3$

⇒ Hallar segmento de $(1,1,1)$ a $(2,1,2)$

dirección del segmento : $\check{v} = (2,1,2) - (1,1,1) = (1,0,1)$

$$\text{segmento } \vec{g}(t) = t(1,0,1) + (1,1,1) \quad , \quad 0 \le t \le 1$$
$$= (t+1, 1, t+1)$$

⇒ Hallar circulación $\vec{f}$ en segmento $\vec{g}$

$$\int_c \vec{f}\, d\vec{s} = \int_0^1 \vec{f}(\vec{g}(t)) \cdot \vec{g}'(t)\, dt = \int_0^1 (2t+2)\ln(t+1) + (t+1) + 1 \; dt$$

$$\int_0^1 2\mu \ln(\mu) + \mu + 1 \, d\mu = (t+1)^2\ln(t+1) + (t+1) \Big|_0^1 = (4\ln(2) + 2) - 1$$
$$= 4\ln(2) + 1$$

$\mu = t+1$

↳ Hallar componentes necesarios

$$\vec{f}(\vec{g}(t)) = \left(2(t+1)\cdot\ln(t+1),\ 3(t+1),\ (t+1)^2/(t+1) + 1\right)$$

$$\vec{g}\,'(t) = (1,0,1)$$

$$2\int \mu \ln(\mu)\, d\mu + \int \mu \, d\mu + \int 1\, d\mu = 2\left(\frac{\mu^2}{2}\ln(\mu) - \frac{1}{2}\int \mu\, d\mu\right) + \frac{\mu^2}{2} + \mu$$
$$= \mu^2\ln(\mu) - \frac{\mu^2}{2} + \frac{\mu^2}{\mu} + \mu$$
$$= (t+1)^2\ln(t+1) + (t+1)$$

$x = \ln(\mu) \quad x' = \frac{1}{\mu}$

$v' = \mu \to v = \frac{\mu^2}{2}$

* Ejercicio 30 : Verificar que circulación no depende de C, sino de puntos iniciales y final de curva
↓
calc integral en (1,3) a (2,4)

$$\int_C \underbrace{(3x-2y^2)\,dx + (y^3-4xy)\,dy}_{\vec{F}\cdot d\vec{s}}$$

$\vec{F}(x,y) = (3x-2y^2,\ y^3-4xy)$ $ds = \frac{ds}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)dt = (dx, dy)$

$$\int_C \vec{F}\cdot d\vec{s}$$

⇒ Verificar si $\vec{F}$ campo de gradientes (conservativo)

↳ Dom $\vec{F} = \mathbb{R}^2$, simplemente conexo

↳ chequear D_F simétrico → chequear D_F simétrico

⇒ Como F es campo conservativo entonces no depende de la trayectoria sino del punto inicial y final

$$D_F \begin{pmatrix} \cdot & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \cdot \end{pmatrix} \quad \frac{\partial P}{\partial y} = -4y \quad \frac{\partial Q}{\partial x} = -4y$$

⇒ calcular circulación (1,3) a (2,4)

$$\int_C (3x-2y^2)\,dx + (y^3-4xy)\,dy = \phi(x,y)\Big|_{(1,3)}^{(2,4)} = \phi(2,4) - \phi(1,3) = 6 - \frac{15}{4} = \frac{9}{4}$$

⇒ Hallar potencial $\phi(x,y) \,/\, \vec{F}(x,y) = \nabla\phi(x,y)$

$$\phi = \int 3x - 2y^2\,dx = \frac{3x^2}{2} - 2y^2x + \alpha(y)$$

$$\frac{\partial\phi}{\partial y}\left(\frac{3x^2}{2} - 2y^2x + \alpha(y)\right) = -4yx + \alpha'(y) = y^3 - 4xy$$

$$\alpha'(y) = y^3 \longrightarrow \alpha(y) = \frac{y^4}{4}$$

$$\phi(x,y) = \frac{3x^2}{2} - 2y^2x + \frac{y^4}{4}$$

$\phi(2,4) = 6$

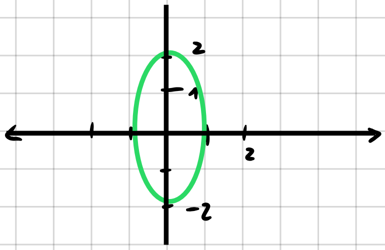
$\phi(1,3) = 15/4$

* **Ejercicio 31:** Evaluar circulación a lo largo de elipse $4x^2+y^2=4$

$$\int_C \left(e^x \operatorname{sen}(y)+3y\right)dx + \left(e^x\cos(y)+2x-2y\right)dy$$

↳ campo vectorial = $\vec{f}(x,y) = \left(e^x\operatorname{sen}(y)+3y,\ e^x\cos(y)+2x-2y\right)$

↳ parametrizar elipse $\quad 4x^2+y^2=4 \downarrow x^2+\frac{y^2}{4}=1$



⇒ si f es conservativo entonces no importa si está en elipse, siempre va a ser

$$\int_C f\cdot ds = \phi(A)-\phi(B) \quad \text{donde } A, B \in \text{elipse}$$

ni tampoco importaría el sentido de circulación

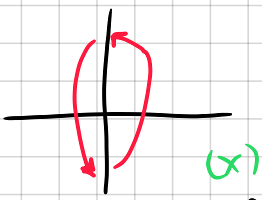
⇒ Chequear $\vec{f}$ conservativo

$\operatorname{Dom}\vec{f} = \mathbb{R}^2 \to$ simplemente conexo

$$D_f = \begin{pmatrix} P'_x & P'_y \\ Q'_x & Q'_y \end{pmatrix} \qquad \left.\begin{aligned} P'_y &= e^x\cos(y)+3 \\ Q'_x &= e^x\cos(y)+2 \end{aligned}\right]$$

como Df no es simétrico entonces no es conservativo

Calcular circulación



(x)

$$\int_{C_+} f\,ds = \int_{C_1} f\,ds + \int_{C_{2+}} f\,ds$$

parametrizar una elipse

$x^2+y^2=4$

$|x| = \sqrt{4-y^2}$

$\vec{g}_1(t) = \left(\sqrt{4-t^2},\ t\right), \quad t\in[-1,1]$

$\vec{g}_1'(t) =$

$\vec{g}_2(t) = \left(-\sqrt{4-t^2},\ -t\right) \quad t\in[-1,1]$

$\vec{g}_2'(t) =$

$x^2+\frac{y^2}{4}=1$

$\to \vec{g}(t) = (\cos(t),\ 2\operatorname{sen}(t)), \quad t\in[0,2\pi]$

$$\int_C f\,ds = \int_0^{2\pi} \vec{f}(\vec{g}(t))\cdot\vec{g}'(t)\,dt$$

$\vec{f}(\vec{g}(t)) = \left(e^{\cos t}\operatorname{sen}(2\operatorname{sen} t)\right.$

$f = g+h$

$$\oint_C f\,ds = \oint g+h\,ds = \underbrace{\oint g\,ds}_{\text{calcular esta}} + \underbrace{\oint h\,ds}_{\text{como es cons, es nulo}}$$

(x1)