ECUACIONES DIFERENCIALES

* Ejercicio 5

Hallar $y'+2xy=4x$ que pasa por $(0,3)$

⇒ Resolver y hallar solución general

$$y'+2xy=4x$$
$$y'=4x-2xy$$
$$y'=2x(2-y)$$
$$y'/(2-y)=2x$$

$$\frac{1}{(2-y)}\cdot\frac{dy}{dx}=2x$$
$$\frac{1}{2-y}dy=2x\,dx$$
$$\int\frac{1}{2-y}dy=\int 2x\,dx$$

$$\ln(|2-y|)=x^2+c$$
$$|2-y|=e^{x^2}\cdot\underbrace{e^c}$$ donde $e^c>0$
$$2-y=\pm e^c e^{x^2} \qquad k=\pm e^c,\ k\in\mathbb{R}$$
$$2-y=ke^{x^2},\ k\in\mathbb{R}$$
$$y=2-ke^{x^2},\ k\in\mathbb{R}$$

⇒ Hallar solución particular (despejar k)

$$3=2-k.e^0=2-k \Rightarrow 2-k=3 \Rightarrow k=-1 \Rightarrow y=2+e^{x^2}$$

* Ejercicio 6

Hallar $y'+x^{-1}y=3x$ que pasa por $(1,4)$

⇒ Hallar solución general

$$y'+x^{-1}y=3x,\quad P(x)=\frac{1}{x}\quad Q(x)=3x$$
$$y'e^{\ln(|x|)}+x^{-1}ye^{\ln(|x|)}=3x.e^{\ln(|x|)} \qquad \int P(x)dx=\ln(|x|)$$
$$\left(ye^{\ln(x)}\right)'=3xe^{\ln(|x|)} \qquad e^{\ln(|x|)}=|x|\to\boxed{x>0}$$

$$(yx)'=3x^2\Rightarrow yx=\int 3x^2dx$$
$$y.x=\frac{3x^3}{3}+c=x^3+c$$
$$y=\frac{x^3}{x}+\frac{c}{x}=x^2+k,\ k\in\mathbb{R}$$

⇒ Hallar solución particular (despejar k)

$$4=1^2+\frac{k}{1}\Rightarrow k=3 \longrightarrow y=x^2+\frac{3}{x}$$

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* Ejercicio 7 Resolver ec diferenciales

(a) $x\,dy=dx,\quad y(-1)=3$

$$1.dy=\frac{1}{x}dx \qquad y(-1)=\underbrace{\ln(|-1|)}_{0}+\overset{3}{C}=3$$
$$y=\ln(|x|)+C \qquad y=\ln(|x|)+3$$

(b) $x.\frac{dy}{dx}-y^2=xy^2$

$$x\frac{dy}{dx}=xy^2+y^2=(x+1)y^2$$
$$\frac{1}{y^2}dy=\frac{(x+1)}{x}.dx$$

$$\int\frac{1}{y^2}dy=\int\left(1+\frac{1}{x}\right)dx \qquad \int y^{-2}dy$$
$$\frac{-1}{y}=x+\ln(|x|)+C$$
$$y=\frac{-1}{x+\ln(|x|)+C}$$

ⓒ $y' + 3y = 2 \qquad P(x) = 3,\ Q(x) = 2$

$y'e^{3x} + 3ye^{3x} = 2e^{3x} \qquad \int P(x)dx = 3x$

$(ye^{3x})' = 2e^{3x}$

$ye^{3x} = 2\int e^{3x}dx \qquad \mu = 3x,\ d\mu = 3dx$

$ye^{3x} = \frac{2}{3}\int e^{\mu}d\mu$

$ye^{3x} = \frac{2}{3}e^{3x} + C$

$y = \frac{2}{3} + \frac{C}{e^{3x}} = \frac{2}{3} + Ce^{-3x}$

ⓓ $y \cdot y' = x\,\text{sen}(x^2)$

$y\,dy = x\,\text{sen}(x^2)\,dx$

$\int y\,dy = \int x\,\text{sen}(x^2)\,dx$

$\frac{y^2}{2} = -\frac{\cos(x^2)}{2} + C$

$y^2 = -\cos(x^2) + K, \quad K \in \mathbb{R}$

ⓔ $y' + y\,\text{sen}(x) = \text{sen}(2x), \quad P(x) = \text{sen}(x),\ Q(x) = \text{sen}(2x)$

$y'e^{-\cos x} + y\,\text{sen}(x)e^{-\cos x} = \text{sen}(2x)\,e^{-\cos x} \qquad \int P(x)dx = -\cos(x)$

$(ye^{-\cos x})' = \text{sen}(2x)\,e^{-\cos x}$

$ye^{-\cos x} = \int \text{sen}(2x)\,e^{-\cos x}dx$

$ye^{-\cos x} = 2\int \text{sen}x\cos x\, e^{-\cos x}dx, \quad \mu = \cos x,\ d\mu = -\text{sen}x\,dx$

$ye^{-\cos x} = -2\int \mu e^{-\mu}d\mu$

$ye^{-\cos x} = -2\left(-\mu e^{-\mu} - \int -e^{-\mu}d\mu\right) \qquad \mu = \mu \to \mu' = 1,\ v' = e^{-\mu} \to v = -e^{-\mu}$

$ye^{-\cos x} = -2\left(-\cos x\, e^{-\cos x} + e^{-\cos x}\right) + C$

$y = -2(-\cos x + 1) + \frac{C}{e^{-\cos(x)}} \to Ce^{\cos x}$

ⓕ $xy' = xy - x, \quad y(1) = 2$

$y' = y - 1$

$\frac{1}{y-1}dy = 1 \cdot dx$

$\ln(|y-1|) = x + C$

$|y-1| = e^x . e^C$

$y - 1 = ke^x$

$y = ke^x + 1$

$2 = ke^1 + 1$

$\boxed{k = \frac{1}{e}} \to y = e^{x-1} + 1$

$y = ke^x + 1$

$y' = ke^x$

ⓖ $y' + 2x^2y = x^2, \quad y(0) = 2$

$y' = x^2 - 2x^2y$

$y' = x^2(1 - 2y)$

$\int \frac{1}{1-2y}dy = \int x^2dx$

$\ln(|1-2y|) = \frac{x^3}{3} + C$

$|1-2y| = e^{\frac{x^3}{3}}.e^C \quad (e^C > 0)$

$1 - 2y = \pm e^C e^{x^3/3} = ke^{\frac{x^3}{3}}$

$y = \frac{1 - ke^{\frac{x^3}{3}}}{2}$

$y(0) = \frac{1 - ke^0}{2} = 2$

$1 - k = 4 \to \boxed{k = -3}$

$y = \frac{1 + 3e^{\frac{x^3}{3}}}{2}$

ⓗ $y' + y = 1, \quad y(0) = \frac{5}{2}$

$y'e^x + ye^x = e^x \qquad P(x) = 1 \quad Q(x) = 1$

$(ye^x)' = e^x \qquad \int P(x)dx = x$

$ye^x = \int e^x dx = e^x + C$

$y = 1 + \frac{C}{e^x}$

$y(0) = 1 + \frac{C}{e^0} = 1 + C = \frac{5}{2}$

$C = \frac{5}{2} - 1 = \frac{3}{2}$

$\boxed{y = 1 + \frac{3}{2e^x}}$

(I) $xy' + y = x^2$, $y(3) = 0$

$(x.y)' = x^2$

$xy = \int x^2 dx = \frac{x^3}{3} + c$

$y = \frac{x^2}{3} + \frac{c}{x}$

$y(3) = \frac{3^2}{3} + \frac{c}{3} = 3 + \frac{c}{3} = 0 \Rightarrow c = -9$ $\rightarrow$ $y = \frac{x^2}{3} - \frac{9}{x}$

(j) $2x\,dx + x^2 y^{-1} dy = 0$

$\frac{x^2}{y} dy = -2x\,dx$

$\frac{1}{y} dy = \frac{-2x}{x^2} dx = \frac{-2}{x} dx$

$\int \frac{1}{y} dy = -2 \int \frac{1}{x} dx$

$\ln|y| = \ln|x|^{-2} + c$

$|y| = |x|^{-2} + e^c$

$y = \pm x^{-2} + k$, $k \in \mathbb{R}, x \in \mathbb{R}$

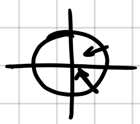
$\boxed{27/5}$

* Ejercicio 8 : Hallar familias curvas donde cada punto

(a) recta normal en cada punto pasa por origen de coord

↳ una curva cualquiera puede ser denotada como $y(x)$

⇒ fig análisis ↳ una recta normal a esa curva en un punto puede ser



$\vec{g}(x) = -\frac{1}{y'} \cdot x + y$

↳ y como tiene que pasar por $(0,0)$ entonces

$\vec{g}(x) = -\frac{1}{y'} \cdot 0 + y = 0$

$\frac{-1}{y'} x + y = 0$

$y = x / y'$

$y \cdot dy = x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

$y^2 = x^2 + C$ → ~~hea~~

↓

(Rehacer) $y = \frac{-1}{y'} \cdot x \rightarrow x^2 + y^2 = c$

(b) recta tg pasa por origen de coord

ecuación de recta tg

$y - y_0 = m(x - x_0)$ $\}$ $(y_0, x_0) = (0,0)$

$y = y'x$

$\frac{1}{y} dy = \frac{1}{x} dx$

$\ln(|y|) = \ln(|x|) + c$

$|y| = e^{\ln(|x|)} \cdot e^c$

$y = |x| . k$, $k \in \mathbb{R}$

$y = kx$

c) recta tg en el punto tiene ordenada al orig igual al doble de ordenada del punto

recordar ecuaciones de rectas $\Rightarrow$ $y - y_0 = m(x - x_0)$, $y = mx + b$

↳ recta tg = $y = y'x + \underbrace{b}$ → ordenada al origen

↳ doble de ordenada al punto $\boxed{b = 2y}$

$y = y'x + 2y$

$y - 2y = \frac{dy}{dx}\,x$

$\frac{1}{(y-2y)}\,dy = \frac{1}{x}\,dx$

$-\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$-\ln|y| = \ln|x| + C$

$|y|^{-1} = |x| \cdot K$

$\frac{1}{y} = \pm xK$

$y = \frac{1}{xK}$, $K \in \mathbb{R}$

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d) recta tg en punto tiene ord al origen igual a la suma de coordenadas en el punto

↳ recta tg = $y - y_0 = y'(x - x_0)$ $\quad y = y'x + b$

↳ suma de coords en el punto = $(y + x)$ → coord (x, y)

↳ resolver eds

→ suponiendo $x \neq 0$

$y = y'x + y + x$

$0 = x(y' + 1)$ → $y' + 1 = 0$

$dy = -1\,dx$

$y = -x + C$

e) recta tg en punto tiene absisa al origen cuatro veces absisa del punto

$y = y'x + 4x$

$y'x - y = -4x$

$y' - \frac{1}{x}y = -4$

$y' \cdot e^{-\ln x} - \frac{1}{x}\,y\,e^{-\ln x} = -4\,e^{-\ln x}$

$P(x) = -\frac{1}{x}$ $\quad Q(x) = -4$

$\int P(x)\,dx = -\ln|x|$

$\left(y \cdot e^{-\ln x}\right) = \int -4\,e^{-\ln x}\,dx$

$y \cdot \left(\frac{1}{x}\right) = -4\ln|x| + C$

$y = -4x\ln|x| + Cx$

FAMILIA DE CURVAS ORTOGONALES

* **Ejercicio 9**: Hallar fam de curvas ortogonales

ⓐ $y = Cx^2$

$$\left(\frac{y}{x^2}\right)' = (C)'$$

$$\frac{y'.x^2 - 2xy}{x^4} = 0$$

$$\Downarrow$$

$$y'x^2 - 2xy = x(y'x - 2y) = 0$$

$$y'x - 2y = 0 \quad \Big\downarrow \text{ cambiar } y' \text{ por } \frac{-1}{y'}$$

$$\frac{-x}{y'} - 2y = 0$$

$$-x = 2y.y'$$

$$y'.x^2 - 2xy = 0$$

$$x(y'x - 2y) = 0$$

$$y'x - 2y = 0$$

$$y'x = 2y$$

$$\frac{1}{y}\,dy = 2.\frac{1}{x}\,dx$$

$$\int \frac{1}{y}\,dy = 2\int \frac{1}{x}\,dx$$

$$\ln|y| = 2\ln|x| + c$$

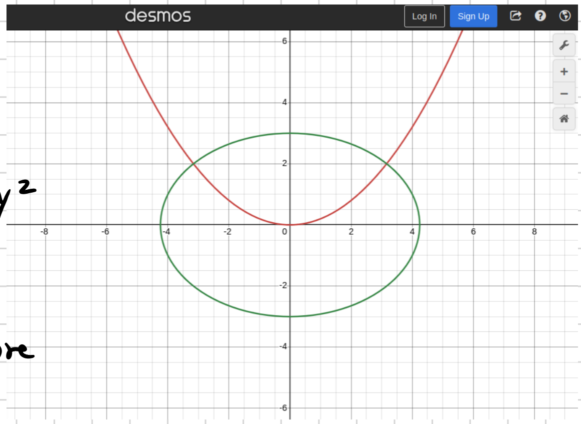
$$y = e^{\ln|x|^2}.k$$

$$y = kx^2$$

$$-\int x\,dx = 2\int y\,dy$$

$$-\frac{x^2}{2} + C = y^2$$

$$\underbrace{y^2 + \frac{x^2}{2} = C}_{\text{elipse}}$$



ⓑ $xy = C$

$$y + y'x = 0 \quad \Big\downarrow \text{ cambiar } y' \text{ por } -y'^{-1}$$

$$y - \frac{x}{y'} = 0$$

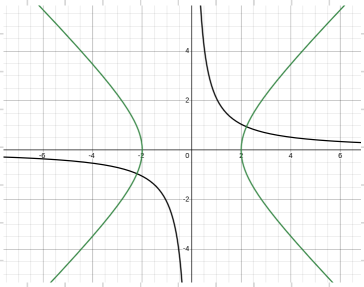
$$y\,y' = x$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y^2 = x^2 + 2C, \quad k \in \mathbb{R} = 2C$$

$$\boxed{-x^2 + y^2 = k} \to \text{hiperbola}$$



ⓒ $x^2 + y^2 = k$

$$2x + 2yy' = 0 \quad \Big\downarrow \text{ cambiar}$$

$$2x - 2y.\frac{1}{y'} = 0$$

$$2x = 2y.\frac{1}{y'}$$

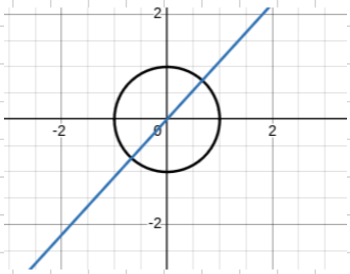
$$xy' = y$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\ln|y| = \ln|x| + c$$

$$|y| = k|x|$$

$$y = kx$$



ⓓ $2x + y = C$

$$2 + y' = 0 \quad \Big\downarrow \text{ camb}$$

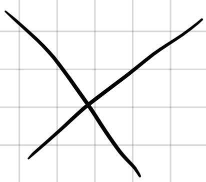
$$2 - \frac{1}{y'} = 0$$

$$2y' - 1 = 0$$

$$2\,dy = 1\,dx$$

$$2y = x + c$$

$$\underbrace{y = \frac{1}{2}x + c}_{\text{recta ortogonal}}$$



ⓔ $x-3 = By^2$

$\dfrac{x-3}{y^2} = B$

$\left(\dfrac{x-3}{y^2}\right)' = (B)'$

$\dfrac{y^2 - 2yy'(x-3)}{y^4} = 0$

$y\left(y - 2y'(x-3)\right) = 0 \quad \rceil\; y=0$

$y - 2y'(x-3) = 0$

$y = 2y'(x-3)$] cambio

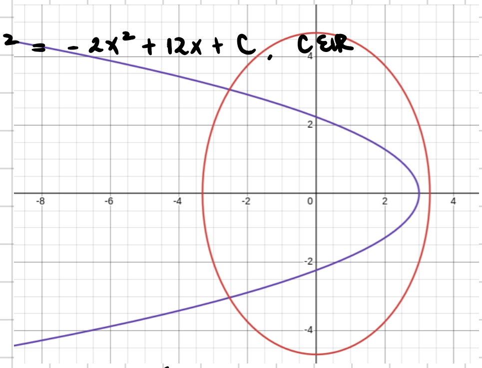
$y = \dfrac{-2(x-3)}{y'}$

$yy' = -2x+6$

$\int y\,dy = -2\int x\,dx + \int 6\,dx$

$\dfrac{y^2}{2} = -x^2 + 6x + C$

$y^2 = -2x^2 + 12x + C, \; C \in \mathbb{R}$



circunferencia con recta

ⓕ $y-1 = Kx$

$\dfrac{y-1}{x} = K$

$\dfrac{y'x - (y-1)}{x} = 0$

$y'x - y + 1 = 0$) cambio

$-\dfrac{x}{y'} - y + 1 = 0$

$\dfrac{x}{y'} = -y+1$

$-\int (y-1)\,dy = \int x\,dx$

$-\dfrac{y^2}{2} + y = \dfrac{x^2}{2} + C$

$y^2 - 2y = -x^2 + C, \; C \in \mathbb{R}$

Líneas de campo $\quad \vec{g}'(t) = \vec{F}(\vec{g}(t))$] → "la mejor ruta"

* Ejercicio 10 = verificar si $\vec{g}(t)$ es línea de campo

ⓐ $\vec{F}(x,y,z) = (y+1,\ 2,\ 1/(2z))$ $\quad \vec{g}(t) = (t^2, 2t-1, \sqrt{t})$, $t>0$

$\vec{F}(g(t)) = (2t,\ 2,\ 1/2\sqrt{t})$, $\quad \vec{g}'(t) = (2t,\ 2,\ \frac{1}{2\sqrt{t}})$, $t>0$

↳ g es línea de campo

ⓑ $\vec{g}(t) = (t^{-3}, e^t, t^{-1})$ $\quad \vec{F}(x,y,z) = (-3z^4, y, -z^2)$

$\vec{F}(\vec{g}(t)) = (-3t^{-4}, e^t, -t^{-2})$ $\quad \vec{g}'(t) = (-3t^{-4}, e^t, -1\,t^{-2})$ $\quad \vec{g}$ es línea de campo

ⓒ $\vec{g}(t) = (\operatorname{sen}(t), \cos(t), e^t)$. $\quad \vec{F}(x,y,z) = (y, -x, z)$

$\vec{F}(\vec{g}(t)) = (\cos(t), -\operatorname{sen}(t), e^t)$ $\quad g'(t) = (\cos(t), -\operatorname{sen} t, e^t)$ $\quad \vec{g}$ es línea de campo

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* Ejercicio 11 Hallar expresión líneas de campo → dado un campo vectorial. línea de campo es óptimo para dejarse llevar por el entorno. trayectoria con misma línea que camp vect

ⓐ $F(x,y) = (-y, x)$ $\quad \vec{g}(t) = (x(t), y(t))$, $\vec{g}'(t) = (x', y')$

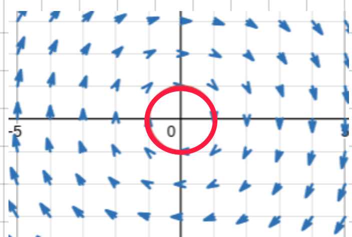
$\begin{cases} -y(t) = x'(t) \\ x(t) = y'(t) \end{cases} \Rightarrow \begin{cases} -y = x' \\ x = y' \end{cases} \Rightarrow \begin{cases} -y = \frac{dx}{dt} \\ x = \frac{dy}{dt} \end{cases} \Rightarrow \dfrac{-dx}{y} = \dfrac{dy}{x}$

$y\,dy = -x\,dx$

$\dfrac{y^2}{2} = -\dfrac{x^2}{2} + C$

$y^2 = -x^2 + C$

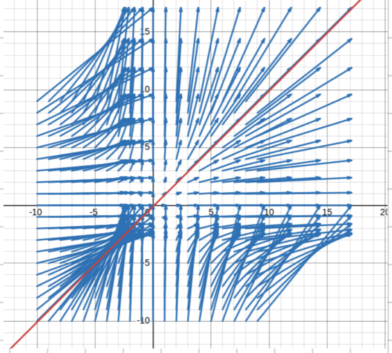
$y^2 + x^2 = C \rightarrow$ circunferencia



(b) $\vec{F}(x,y) = (x^2, y^2)$

$$\frac{dx}{x^2} = \frac{dy}{y^2} \qquad -y^{-1} = -x^{-1} + C$$

$$\int y^{-2}\,dy = \int x^{-2}\,dx \qquad \frac{1}{y} = \frac{1}{x} - C \Rightarrow y = x - \frac{1}{C}$$



(c) $F(x,y) = \left(\frac{1}{2x-y}, \frac{1}{x}\right)$ $\qquad P(x) = \frac{1}{x} \qquad -\int P(x)\,dx = -\ln|x|$

$(2x-y)\,dx = x\,dy$

$2x - y = x y'$

$y = 2x - xy'$

$\frac{y}{x} = 2 - y'$

$y' = 2 - \frac{y}{x}$

$y' - \frac{y}{x} = 2$

$y' e^{-\ln|x|} - \frac{y}{x} \cdot e^{-\ln|x|} = 2e^{-\ln|x|}$

$y \frac{1}{x} = 2\int \frac{1}{x}\,dx$

$y \cdot \frac{1}{x} = 2\ln|x| + C$

$y = 2x\ln|x| + xC$

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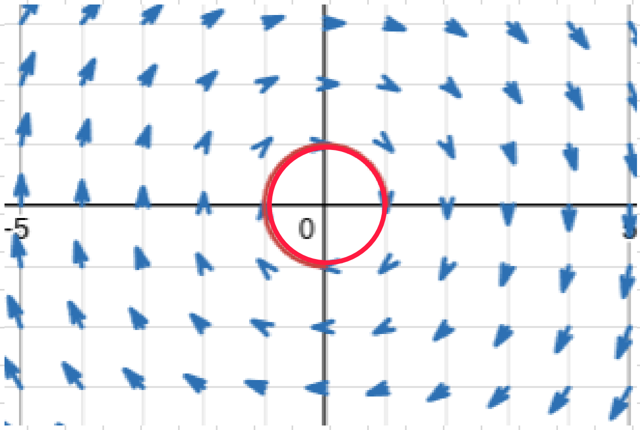
(d) $\vec{F}(x,y) = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2}\right)$



$$\frac{dx}{\left(\frac{y}{x^2+y^2}\right)} = \frac{dy}{\left(\frac{-x}{(x^2+y^2)}\right)}$$

$$\frac{dx}{y} = \frac{dy}{-x}$$

$y\,dy = -x\,dx$

$\frac{y^2}{2} = \frac{-x^2}{2} + C$

$y^2 + x^2 = C$

(e) $\vec{F}(x,y,z) = (x, y^2, z)$ $\qquad$ Hallar $\vec{g}(t)$ donde $\vec{F}(\vec{g}(t)) = \vec{g}'(t)$

$$\begin{cases} x = x' \\ y^2 = y' \\ z = z' \end{cases} \Rightarrow \begin{cases} x = \frac{dx}{dt} \\ y^2 = \frac{dy}{dt} \\ z = \frac{dz}{dt} \end{cases} \Rightarrow \frac{dx}{x} = \frac{dy}{y^2} = \frac{dz}{z}$$

$$\int \frac{1}{x}\,dx = \int y^{-2}\,dy = \int \frac{1}{z}\,dz$$

$$\ln|x| = -y^{-1} + C = \ln|z|$$

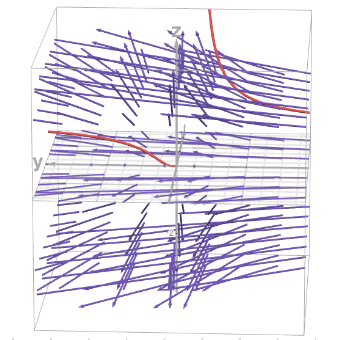
$$|x| = e^{-\frac{1}{y}} \cdot e^{C} = |z|$$

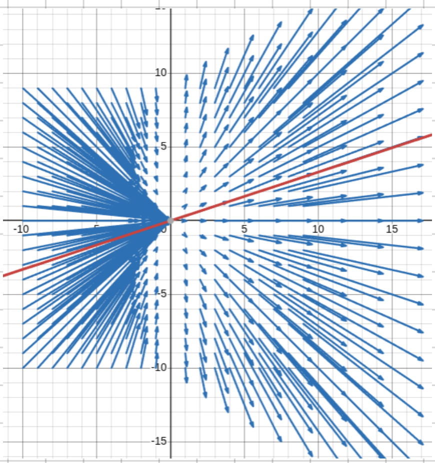
$x = \pm e^{C} e^{-\frac{1}{y}} = k e^{-\frac{1}{y}}$

$z = \pm e^{C} e^{-\frac{1}{y}} = k e^{-\frac{1}{y}}$

$\Downarrow$

$\vec{g}(t) = \left(k e^{-1/t}, t, k e^{-1/t}\right)$





(f) $\vec{F}(x,y) = (x^2, xy)$

$$\frac{dx}{x^2} = \frac{dy}{xy}$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$\ln|y| = \ln|x| + C$

$|y| = e^{\ln|x|} \cdot e^{C}$

$y = kx$