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POLINOMIO DE TAYLOR

* *Ejercicio 2* Obtener polinomio Taylor 2° orden f en A

(a) $f(x,y) = e^{x+y}\cos(y-1)$, $A = (-1, 1) \to (x_0, y_0)$ $\qquad f(-1,1) = e^0 \cos(0) = 1$

- Hallar derivadas primeras

$\frac{\partial f}{\partial x}(x,y) = e^{x+y}\cos(y-1) \longrightarrow \frac{\partial f}{\partial x}(-1,1) = 1$

$\frac{\partial f}{\partial y}(x,y) = e^{x+y}\cos(y-1) + (-\mathrm{sen}(y-1) \cdot e^{x+y}) \longrightarrow \frac{\partial f}{\partial y}(-1,1) = 1$

- Hallar derivadas segundas

$\frac{\partial^2 f}{\partial x^2}(x,y) = e^{x+y}\cos(y-1) \longrightarrow \frac{\partial^2 f}{\partial x^2}(-1,1) = 1$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = e^{x+y}\cdot\cos(y-1) - \mathrm{sen}(y-1)\, e^{x+y} \longrightarrow \frac{\partial^2 f}{\partial y \partial x}(-1,1) = 1$

$\frac{\partial^2 f}{\partial y^2}(x,y) = \left[e^{x+y}\cos(y-1) - \mathrm{sen}(y-1)e^{x+y}\right] - \left[\cos(y-1)e^{x+y} + e^{x+y}\mathrm{sen}(y-1)\right] \longrightarrow \frac{\partial^2 f}{\partial y^2}(-1,1) = 0$

$$P_2(x) = f(-1,1) + \nabla f(-1,1)\,(x+1,\ y-1) + \frac{1}{2!}\left(\frac{\partial^2 f}{\partial x^2}(x+1)^2 + \frac{\partial^2 f}{\partial y^2}(y-1)^2\right) + \frac{\partial^2 f}{\partial y \partial x}(-1,1)\,(x+1)(y-1)$$

$$= 1 + (x+1) + (y-1) + \frac{1}{2}\left((x+1)^2 + 0(y-1)^2\right) + 1\,(x+1)(y-1)$$

$$= 1 + (x+1) + (y-1) + \frac{1}{2}(x+1)^2 + (x+1)(y-1)$$

(b) $f(x,y) = \cos(x+y)$, $A = (0,0)$ $\qquad f(0,0) = 1$

- Hallar primeras derivadas

$\frac{\partial f}{\partial x}(x,y) = -\mathrm{sen}(x+y) \longrightarrow \frac{\partial f}{\partial x}(0,0) = 0$ $\qquad \frac{\partial f}{\partial y}(x,y) = -\mathrm{sen}(x+y) \longrightarrow \frac{\partial f}{\partial y}(0,0) = 0$

- Hallar segundas derivadas

$\frac{\partial^2 f}{\partial x^2}(x,y) = -\cos(x+y) \longrightarrow \frac{\partial^2 f}{\partial x^2}(0,0) = -1$ $\qquad \frac{\partial^2 f}{\partial y^2}(x,y) \to -\cos(x+y) \longrightarrow \frac{\partial^2 f}{\partial y^2}(0,0) = -1$

$\frac{\partial^2 f}{\partial y \partial x}(x,y) = -\cos(x+y) \to \frac{\partial^2 f}{\partial y \partial x}(0,0) = -1$

$$P_2(x) = f(0,0) + \frac{\partial f}{\partial x}(0,0)\,(x) + \frac{\partial f}{\partial x}(y) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(0,0)\,(x)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial y^2}(0,0)\,y^2 + \frac{\partial^2 f}{\partial y \partial x}\,xy$$

$$= 1 - \frac{1}{2}x^2 - \frac{1}{2}y^2 - xy$$

(c) $f(x,y,z)=\sqrt{xy}\,\ln(z)$ en $A=(1,4,1)$ $\quad f(A)=0$

- Hallar derivadas primeras

$$\frac{\partial f}{\partial x}(x,y,z)=\frac{1}{2\sqrt{xy}}\cdot y\cdot\ln(z)\longrightarrow\frac{\partial f}{\partial x}(1,4,1)=\frac{1}{2\sqrt{4}}\cdot 4\cdot\ln(1)=0$$

$$\frac{\partial f}{\partial y}(x,y,z)=\frac{1}{2\sqrt{xy}}\,x\ln(z)\longrightarrow\frac{\partial f}{\partial y}(1,4,1)=\frac{1}{2\sqrt{4}}\,1\cdot\ln(1)=0$$

$$\frac{\partial f}{\partial z}(x,y,z)=\sqrt{xy}\,\frac{1}{z}\longrightarrow\frac{\partial f}{\partial z}(1,4,1)=\sqrt{4}\cdot\frac{1}{1}=2$$

- Hallar derivadas segundas

$$\frac{\partial^2 f}{\partial x^2}(x,y,z)=\frac{1}{2}\ln(z)\,y\,\frac{\partial}{\partial x}(xy)^{-\frac{1}{2}}=-\frac{1}{4}\ln(z)\,y^2\,\frac{1}{\sqrt{(xy)^3}}\longrightarrow\frac{\partial^2 f}{\partial x^2}(1,4,1)=\frac{-1}{4}\ln(1)\frac{4^2}{\sqrt{4^3}}=0$$

$$\frac{\partial^2 f}{\partial y^2}(x,y,z)=\frac{-1}{4}\ln(z)\,x^2\frac{1}{\sqrt{(xy)^3}}\longrightarrow\frac{\partial^2 f}{\partial y^2}(1,4,1)=0$$

$$\frac{\partial^2 f}{\partial x\partial y}(x,y,z)=\frac{\ln(z)}{2}\,\frac{\partial}{\partial y}\left(\frac{y}{\sqrt{xy}}\right)=\ln(z)\frac{1}{2}\left(\frac{\sqrt{xy}+\frac{1}{2\sqrt{xy}}\cdot y}{xy}\right)\longrightarrow\frac{\partial^2 f}{\partial x\partial y}(A)=0$$

$$\frac{\partial^2 f}{\partial z\partial x}(x,y,z)=\frac{1}{2\sqrt{xy}}\,y\cdot\frac{1}{z}\longrightarrow\frac{\partial^2 f}{\partial z\partial x}(A)=\frac{4}{2\sqrt{4}}\cdot\frac{1}{1}=1$$

$$\frac{\partial^2 f}{\partial z\partial y}(x,y,z)=\frac{1}{2\sqrt{xy}}\,x\cdot\frac{1}{z}\longrightarrow\frac{\partial^2 f}{\partial z\partial y}(A)=\frac{1}{4}\cdot 1=\frac{1}{4}$$

$$\frac{\partial^2 f}{\partial z^2}(x,y,z)=\sqrt{xy}\cdot(z^{-1})'=-\sqrt{xy}\,\frac{1}{z^2}\longrightarrow\frac{\partial^2 f}{\partial z^2}(A)=\frac{-\sqrt{4}}{1}=-2$$

- Armar polinomio

$$\hookrightarrow P_2(x)=f(A)+\nabla f(A)\,(x-1,\,y-4,\,z-1)+\frac{1}{2}\frac{\partial^2 f}{\partial x^2}(A)(x-1)^2+\frac{1}{2}\frac{\partial^2 f}{\partial y^2}(A)(y-4)^2+\frac{1}{2}\frac{\partial^2 f}{\partial z^2}(A)(z-1)^2$$

$$+\frac{\partial^2 f}{\partial y\partial x}(x-1)(y-4)+\frac{\partial^2 f}{\partial z\partial x}(x-1)(z-1)+\frac{\partial^2 f}{\partial y\partial z}(A)(y-4)(z-1)$$

$$=0+(0,0,2)(x-1,y-4,z-1)+0(x-1)^2+0(y-4)^2+\frac{1}{2}(-2)(z-1)^2+0(x-1)(y-4)+(x-1)(z-1)+\frac{1}{4}(y-4)(z-1)$$

$$=2(z-1)-(z-1)^2+(x-1)(z-1)+\frac{1}{4}(y-4)(z-1)$$

* Ejercicio 3   Aproximar valor $1{,}01^{1{,}98}$ usando polinomio Taylor ord 1 en $A=(1,2)$

$f(x,y) = x^y$ → aproxim Taylor en $A=(1,2)$

- $f(A) = 1^2 = 1$
- Hallar derivadas primeras

$\frac{\partial f}{\partial x}(x,y) = y(x)^{y-1} \longrightarrow \frac{\partial f}{\partial x}\Big|_{1,2} = 2(1)^1 = 2$ $\frac{\partial f}{\partial y}(x,y) = \frac{1}{y}\cdot \ln(x) \rightarrow \frac{\partial f}{\partial y}(1,2) = \frac{1}{2}\cdot \ln(1) = 0$

- $P_1(x,y) \cong f(A) + \frac{\partial f}{\partial x}(A)(x-1) + \frac{\partial f}{\partial y}(A)(y-2) = 1 + 2(x-1) + 0(y-2) = 2x-1 + R_1(x)$

$P_1(1{,}01, 1{,}98) = 2.(1{,}01) - 1 = 2{,}02 - 1 \cong 1{,}02$ → $1{,}01^{1{,}98} \cong 1{,}02$

* Ejercicio 4 : Demo $E(P=(1,2))$ $e^{x-1}\ln(y-1) \cong y-2$ → grado 1

$f(x,y) = e^{x-1}\ln(y-1) \longrightarrow f(1,2) = e^0 \ln(1) = 0$

- Hallar derivadas primeras

$P_1(x,y) \cong \underbrace{f(1,2) + \frac{\partial f}{\partial x}(1,2)(x-1)}_{0} + \frac{\partial f}{\partial y}(1,2)(y-2) \cong y-2$

$\frac{\partial f}{\partial x}(x,y) = \ln(y-1)e^{x-1} \longrightarrow \frac{\partial f}{\partial x}(1,2) = 0$ $\frac{\partial f}{\partial y}(x,y) = \frac{1}{y-1}e^{x-1} \rightarrow \frac{\partial f}{\partial y}(1,2) = \frac{1}{1}(e^0) = 1$

* Ejercicio 5 $y - z + e^{zx} = 0$ ec implícita $z = f(x,y)$ pasando por $A=(0,0,1)$ → $\boxed{z = f(0,0) = 1}$

↪ Hallar aprox $f(0{,}01, -0{,}02)$ mediante taylor orden 2 → aproximacion en entorno $(x_0, y_0) = (0,0)$

$$P_2(x,y) \cong f(0,0) + \nabla f(0,0)(x,y) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(0,0)x^2 + \frac{1}{2}\frac{\partial^2 f}{\partial y^2}(0,0)y^2 + \frac{\partial^2 f}{\partial x \partial y}(0,0)xy$$

→ Hallar derivadas primeras

$\frac{\partial}{\partial x}\left(y - f(x,y) + e^{f(x,y)x}\right) = \frac{\partial}{\partial x}(0)$

$-\frac{\partial f}{\partial x}(x,y) + \left[e^{f(x,y)x}\right]\left(\frac{\partial f}{\partial x}(x,y).x + xf(x,y)\right) = 0$

eval (0,0) ↓

$-\frac{\partial f}{\partial x}(0,0) + e^0\left(\frac{\partial f}{\partial x}f(0,0).0 + 0.f(0,0)\right) = 0$

↪ $\frac{\partial f}{\partial x}(0,0) = 0$

$\frac{\partial}{\partial y}\left(y - f(y,x) + e^{f(y,x)x}\right) = \frac{\partial}{\partial y}(0)$

$1 - \frac{\partial f}{\partial y}(x,y) + e^{f(x,y)x}\frac{\partial f(x,y)}{\partial y}.x = 0$

eval (0,0) ↓

$1 - \frac{\partial f}{\partial y}(0,0) + 0 = 0$

$\boxed{\frac{\partial f}{\partial y}(0,0) = 1}$

$P_2(x,y) = 1 + (y) + \frac{1}{2}x^2$

$f(0{,}01, -0{,}02) \cong \underbrace{1 - 0{,}02 + \frac{1}{2}(0{,}01)^2}_{0{,}98}$

→ Hallar derivadas segundas

$\frac{\partial}{\partial x}\left(-\frac{\partial f}{\partial x}(x,y) + e^{f(x,y)x}\left(\frac{\partial f}{\partial x}(x,y)x + xf(x,y)\right)\right) = 0$

$-\frac{\partial^2 f}{\partial x^2}(x,y) + e^{f(x,y)x}\left(\frac{\partial f}{\partial x}(x,y)x + xf(x,y)\right)^2 + \left(\frac{\partial^2 f}{\partial x^2}(x,y)x + \frac{\partial f}{\partial x}(x,y) + f(x,y) + \frac{\partial f(x,y)}{\partial x}x\right)e^{f(x,y)x} = 0$

eval (0,0) ↓

$= -\frac{\partial^2 f}{\partial x^2}(0,0) + e^0(0) + e^0\left(\frac{\partial^2 f}{\partial x^2}(0,0)0 + 0 + 1 + 0\right). = 0$

$\boxed{\frac{\partial^2 f}{\partial x^2} = 1}$

$\frac{\partial f}{\partial y}\left(1 - \frac{\partial f}{\partial y}(x,y) + e^{f(x,y)x}\frac{\partial f(x,y)}{\partial y}x\right) = 0$

$-\frac{\partial^2 f}{\partial y^2}(x,y) + e^{f(x,y)x}\cdot\left(\frac{\partial f(x,y)}{\partial y}x\right)^2 + \frac{x\,\partial^2 f(x,y)}{\partial y^2}e^{f(x,y)x}$

$-\frac{\partial^2 f}{\partial y^2}(0,0) + e^0.0 + 0 = 0$ → $\boxed{\frac{\partial^2 f}{\partial y^2}(0,0) = 0}$

$\frac{\partial}{\partial x}\left(1 - \frac{\partial f}{\partial y}(x,y) + e^{f(x,y)x}\frac{\partial f}{\partial y}(x,y)x\right) = 0$

¿? $-\frac{\partial^2 f}{\partial x \partial y}(x,y) = 0$

* **Ejercicio 6** $\quad p(x,y) = x^2 - 3xy + 2x + y - 1 \to$ taylor ord 2 de f en $(2,1)$

$\hookrightarrow$ Hallar ec cartesiana plano tg de f en $(2,1,z_0)$

$\boxed{\begin{array}{l}\hookrightarrow \text{si p es taylor de f entonces} \\ \nabla p(2,1) = \nabla f(2,1) \ \text{ y } \ p(2,1) = f(2,1)\end{array}}$

$\longrightarrow f(2,1) = p(2,1) = 2$

$\longrightarrow \nabla p(x,y) = (2x - 3y + 2 \, , \, -3x + 1)$

$\nabla p(2,1) = (3 \, , -5)$

$\to$ ec plano tg a f

$$z = f(2,1) + \nabla f(2,1)\,(x-2, y-1)$$

$$z = p(2,1) + \nabla p(2,1)\,(x-2, y-1) \longrightarrow z = 2 + (3,-5)(x-2, y-1) = 3x - 5y + 1 = 0$$

* **Ejercicio 7** $\quad w = f(\mu, v)$ implícita $3v + \mu e^{2w} - w = 1$ con $f(7,-2) = 0$

$\mu = x - 2y \quad v = x + y$ – hallar taylor ord 1 para $w(x,y)$ en $(x,y) = (1,-3)$ y calc $w, \ (x,y) = (0{,}97, -3{,}01)$

$\downarrow$

$w: \mathbb{R}^2 \to \mathbb{R} \,/\, w = f \circ g$, donde $g = (x - 2y \, , \, x + y)$,

$\hookrightarrow p_1(x) = f(g(1,-3)) + \frac{\partial w}{\partial x}(1,-3)(x-1) + \frac{\partial w}{\partial y}(1,-3)(y+3) = \nabla w(1,-3)(x-1, y+3)$

$\to$ Empezar con $g(1,-3)$

$g(1,-3) = (7,-2) \qquad f(g(1,-3)) = f(7,-2) = 0$

$\longrightarrow$ Hallar $\frac{\partial w}{\partial x}$ y $\frac{\partial w}{\partial y}$

$$w \to \mu \to x, y \qquad w \to v \to x, y$$

$$\frac{\partial w}{\partial x}(x,y) = \frac{\partial f}{\partial \mu}(\mu,v)\frac{\partial g}{\partial x}(x,y) \qquad (\mu,v) = g(x,y)$$

$$\frac{\partial w}{\partial y}(x,y) = \frac{\partial f}{\partial \mu}\underbrace{(\mu,v)}_{g(x,y)}\frac{\partial g}{\partial y}(x,y)$$

$$\nabla w(x,y) = \nabla f(\mu,v) \cdot Dg(x,y)$$

$$\nabla w(1,-3) = \nabla f(7,-2) \cdot Dg(1,-3)$$

$$\nabla w(1,-3) = \left(\frac{-1}{6}, \frac{-1}{2}\right)\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} = \left(\frac{-2}{3}, \frac{-1}{6}\right)$$

$\nabla f(\mu,v) = (f'_\mu(7,-2) \, , \, f'_v(7,-2)) \to \nabla f(7,-2) = \left(\frac{-1}{6}, \frac{1}{2}\right)$

∘ Hallar $f'_\mu(7,-2)$

$$\frac{\partial}{\partial \mu}\left(3v + \mu e^{2f(\mu,v)} - f(\mu,v)\right) = \frac{\partial}{\partial \mu}(1)$$

$$e^{2f(\mu,v)} + e^{2f(\mu,v)}\frac{\partial f}{\partial \mu}(\mu,v)\cdot\mu - \frac{\partial f}{\partial \mu}(\mu,v) = 0$$

$$e^{2\cdot 0} + e^{2\cdot 0}\frac{\partial f}{\partial \mu}(7,-2)\cdot 7 - \frac{\partial f}{\partial \mu}(7,-2) = 0$$

$$1 + 6\frac{\partial f}{\partial \mu}(7,-2) = 0 \longrightarrow \frac{\partial f}{\partial \mu}(7,-2) = \frac{-1}{6}$$

Hallar $\frac{\partial f}{\partial v}(7,-2)$

$$3 + \mu e^{2f(\mu,v)}\frac{\partial f}{\partial v}(\mu,v) - \frac{\partial f}{\partial v}(\mu,v) = 0$$

$$3 + 7\frac{\partial f}{\partial v}(7,-2) - \frac{\partial f}{\partial v}(7,-2) = 0$$

$$\frac{\partial f}{\partial v}(7,-2) = \frac{-1}{2}$$

$$p_1(x,y) = \nabla w(1,-3)(x-1, y+3) = \left(\frac{-2}{3}, \frac{-1}{6}\right)(x-1, y+3)$$

$$= \frac{-2}{3}x - \frac{1}{6}y + \frac{1}{6} = 0$$

$$p(0.97, -3.01) \cong \frac{-2}{3}(0{,}97) - \frac{1}{6}(-3.01) + \frac{1}{6} \cong 0{,}021$$

∘ Hallar $Dg(x,y)$

$$Dg(x,y) = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} \longrightarrow Dg(1,-3) = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$

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EXTREMOS

*** Ejercicio 9:** Hallar extremos $f(x,y) = x^2+xy+y^2-ax-by$, $a,b \in \mathbb{R}$

- Hallar $\nabla f(x,y)$

$$\nabla f(x,y) = (2x+y-a \,,\, 2y+x-b)$$

- Hallar $H_f(x_0,y_0)$

$$H_f(x,y) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

- Hallar (x_0,y_0) para $\nabla f(x_0,y_0)=0$ → puntos críticos

$$\begin{cases} 2x+y-a=0 \\ x+2y-b=0 \end{cases} \rightarrow \left(\begin{array}{cc|c} 2 & 1 & a \\ 1 & 2 & b \end{array}\right) \xrightarrow{F_2 \to 2F_2-F_1} \left(\begin{array}{cc|c} 2 & 1 & a \\ 0 & 3 & 2b-a \end{array}\right)$$

$$\begin{cases} 2x+y=a \longrightarrow x = \frac{a-y}{2} = \frac{1}{2}\cdot\left(a-\frac{2b-a}{3}\right) = \frac{1}{2}\left(\frac{4a-2b}{3}\right) \\ 3y=2b-a \longrightarrow y = \frac{2b-a}{3} \end{cases}$$

$$x = \left(\frac{2a-b}{3}\right)$$

Punto crítico $\left(\frac{2a-b}{3}, \frac{2b-a}{3}\right)$

$$\det(H_f(x_0,y_0)) = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$$

como $\det(H_f) > 0$ y $\frac{\partial^2 f}{\partial x^2}(x_0,y_0) > 0$ el punto crítico $\left(\frac{2a-b}{3}, \frac{2b-a}{3}\right)$ es un mínimo

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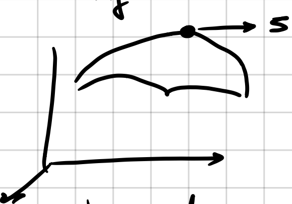
*** Ejercicio 10** Proponer una función con max local en $(1,-2)$ valor 5

fig



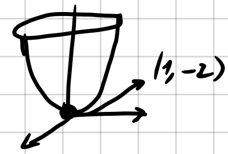
↳ condiciones a cumplir
- $f(1,-2) = 5$
- $\nabla f(1,-2) = (0,0)$ → $\frac{\partial f}{\partial x}(1,-2) = 0$ $\frac{\partial f}{\partial y}(1,-2) = 0$
- $\det(H_f) > 0$
- $\frac{\partial^2 f}{\partial x^2}(1,-2) < 0$

→ alguna función que sea diferenciable y max en $z = f(1,-2) = 5$?

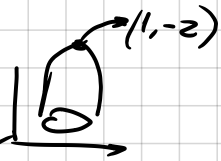
→ Intentar con paraboloide

paraboloide

$$\boxed{z = (x-1)^2 + (y+2)^2}$$



↓ cambiar indinación



$$z = -(x-1)^2 - (y+2)^2 + \underline{5}$$

$$z = -(x^2-2x+1) - (y^2+4y+4) + 5$$

$$z = -x^2+2x-y^2-4y-5+5$$

$z = f(1,-2) = 5$

$$f(x,y) = -x^2+2x-y^2-4y$$

- Verificar paraboloide max $(1,-2)$

$$\nabla f(x,y) = (-2x+2 \,,\, -2y-4)$$

$$\begin{cases} -2x+2=0 \\ -2y-4=0 \end{cases} \rightarrow \text{punto crítico } (1,-2)$$

$$H_f(x,y) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \rightarrow H_f(1,-2) = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$$

como $\det(H_f(1,-2)) > 0$ y $\frac{\partial^2 f}{\partial x^2}(1,-2) < 0$, el punto $(1,-2)$ es max absoluto de $f(x,y) = -x^2+2x-y^2-4y$

* Ejercicio 11 $\quad f(x,y) = ax^3+bxy+cy^2$ → Hallar a,b,c para $(0,0,0)$ sea silla y $f(1,1)$ sea mínimo local

$(x_0\ y_0)=(0,0)$ $\qquad (1,\ 1,\ a+b+c)$

- Hallar $\nabla f(x,y) = (3ax^2+by,\ bx+2cy)$

↳ Buscar ptos críticos

$$\begin{cases} 3ax^2+by=0 \\ bx+2cy=0 \end{cases}$$

Las condiciones a cumplir

→ $\det(H_f(0,0)) < 0$ → no es extremo

→ $\det(H_f(1,1)) > 0$ y $\frac{\partial^2 f}{\partial x^2}(1,1) > 0$ → min local

↳ Buscar $H_f(x,y)$

$$H_f(x,y) = \begin{pmatrix} 6ax & b \\ b & 2c \end{pmatrix} \qquad H_f(0,0) = \begin{pmatrix} 0 & b \\ b & 2c \end{pmatrix} \qquad H_f(1,1) = \begin{pmatrix} 6a & b \\ b & 2c \end{pmatrix}$$

Hallar a,b,c para cumplir condiciones

$\det(H_f(0,0)) < 0 \rightarrow -b^2 < 0 \rightarrow b \neq 0 \quad a>0 \quad a\neq 0$

$\det(H_f(1,1)) > 0 \rightarrow \underbrace{12ac-b^2>0 \ \text{ y } \ 6a>0}_{c>0}$

$\nabla f(1,1) = (0,0)$

$$\begin{cases} 3a+b=0 \rightarrow b=-3a \\ b+2c=0 \rightarrow 3a=2c \rightarrow c=\frac{3a}{2} \end{cases}$$

reemplazo $b=-3a$

$$\begin{cases} -(3a)^2 = -9a^2 < 0 \\ 12ac-9a^2>0 \end{cases}$$

$\rightarrow c > \frac{3}{4}a^2$

$\boxed{a>0}$

$\boxed{b=-3a}$

$\boxed{c=\frac{3a}{2}}$

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* Ejercicio 12 $\quad f>0$ y $f\in C^3 \quad \nabla f(\underbrace{1,-1}_{P_1}) = \nabla f(\underbrace{-1,1}_{P_2}) = (0,0) \quad$ $f(P_1)=10$ max local → $f''_{xx}(P_1)<0$

$f(P_2)=3$ min local → $f''_{xx}(P_2)>0$

↳ estudiar extremos $\quad g(x,y) = 1/f(x,y)$

- Hallar gradiente $\nabla g(x,y)$ → Hallar puntos cuando $\nabla g(x,y)=\vec{0}$

$$\frac{\partial}{\partial x}(1/f(x,y)) = \frac{-1}{(f(x,y))^2}\cdot f'_x(x,y)$$

$$\frac{\partial}{\partial y}(1/f(x,y)) = \frac{-1}{(f(x,y))^2}\cdot f'_y(x,y)$$

$$\begin{cases} f'_x(x,y)=0 \rightarrow f'_x(1,-1)=0,\ f'_x(-1,1)=0 \\ f'_y(x,y)=0 \rightarrow f'_y(1,-1)=0,\ f'_y(-1,1)=0 \end{cases}$$

ptos críticos P_1 y P_2

- Hallar H_g

$$H_g(x,y) = \begin{pmatrix} -\left(\frac{f''_{xx}f(x,y)^2-2f(x,y)f'_x(x,y)^2}{(f(x,y))^4}\right) & -\left(\frac{f_{xy}f(x,y)^2-2f(x,y)f_x(x,y)f_y(x,y)}{(f(x,y))^4}\right) \\ -\left(\frac{f'_{yx}(x,y)(f(x,y))^2-2f(x,y)f_x(x,y)f_y(x,y)}{(f(x,y))^4}\right) & -\left(\frac{f''_{yy}f(x,y)^2-2f(x,y)f'_y(x,y)^2}{(f(x,y))^4}\right) \end{pmatrix}$$

$H_g(P_1) = \begin{pmatrix} >0 & \\ & \end{pmatrix}$ → P_1 es mínimo

$H_g(P_2) = \begin{pmatrix} <0 & \\ & \end{pmatrix}$ → P_2 es max

más fácil pensar denom ↗ → mínimo, min denom → max valor $\frac{1}{a}$

* Ejercicio 15 Hallar extremos locales de $f(x,y) = 27x + y + (xy)^{-1}$

- Hallar $\nabla f(x,y)$

$$\left.\begin{array}{l} \frac{\partial f}{\partial x}(x,y) = 27 + (-1)(xy)^{-2} \cdot y \\ \frac{\partial f}{\partial y}(x,y) = 1 + (-1)(xy)^{-2} \cdot x \end{array}\right\} \nabla f(x,y) = \left(27 - (xy)^{-2} y,\ 1 - (xy)^{-2} x\right)$$

- Hallar puntos críticos $\nabla f(x,y) = \vec{0}$

$$\left\{\begin{array}{l} 27 - (xy)^{-2} \cdot y = 0 \to 27 - \frac{y}{x} = 0 \to \boxed{y = 27x} \\ 1 - (xy)^{-2} x = 0 \to \underbrace{(xy)^{-2}}\, = \frac{1}{x} \to \frac{1}{x^2 y^2} = \frac{1}{x} \end{array}\right.$$

reemplazar $(y2)^2$

$$\to 27 - \frac{1}{x^2} \cdot \frac{1}{y^2} \cdot y = 0$$

$$\boxed{\frac{1}{x} = y^2} \to \boxed{x = \frac{1}{y^2}}$$

$$27 - y^2 \cdot y = 0$$

$$\left.\begin{array}{l} y = \sqrt[3]{27} = 3 \\ x = \frac{1}{3^2} = \frac{1}{9} \end{array}\right\} P_1\left(\frac{1}{9}, 3\right)$$

→ reemplazar en ec_2

$$27 - \frac{1}{(xy)^2} \cdot y = 0$$

$$27 = \frac{1}{x^2} \cdot \frac{1}{y} \to \boxed{y = \frac{1}{27x^2}}$$

$$y = \frac{1}{27\left(\frac{1}{9}\right)^2} = 3$$

$$1 - \frac{1}{(xy)^2} \cdot x = 1 - \frac{1}{x} \cdot \frac{1}{y^2} = 1 - \frac{1}{x} \cdot \left(\frac{1}{\frac{1}{27x^2}}\right)^2 = 1 - \frac{1}{x}(27x^2)^2 = 0 \to 1 = 729x^3$$

$$x = \frac{1}{9}$$

$$\boxed{P = \left(\frac{1}{9}, 3\right)}$$

- Verificar extremos

$$\frac{\partial^2 f}{\partial x^2}(x,y) = 2y(xy)^{-3} \cdot y \qquad \frac{\partial^2 f}{\partial y^2} = 2x(xy)^{-3} \cdot x \qquad H_f(x,y) = \begin{pmatrix} 2y(xy)^{-3} y & \frac{1}{(xy)^2} \\ \frac{1}{(xy)^2} & 2x^2(xy)^{-3} \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial}{\partial x}\left(1 - (xy)^{-2} x\right)$$

$$- \left[-2(xy)^{-3} \cdot x \cdot y + (xy)^{-2}\right]$$

$$- \left[-\frac{2}{(xy)^3} xy + \frac{1}{(xy)^2}\right] = \frac{2}{(xy)^2} - \frac{1}{(xy)^2} = \frac{1}{(xy)^2}$$

$$H_f\left(\frac{1}{9}, 3\right) = \begin{pmatrix} 486 & 9 \\ 9 & \frac{2}{3} \end{pmatrix}$$

$\det\left(H_f\left(\frac{1}{9}, 3\right)\right) = 243 > 0 \to$ como $\det\left(H_f\left(\frac{1}{9}, 3\right)\right) > 0$ y $\frac{\partial^2 f}{\partial x^2} > 0$ entonces $P = \left(\frac{1}{9}, 3\right)$ es mínimo local

$\boxed{4/5}$

* Ejercicio 16 Demostrar $f(x,y,z) = 4xyz - x^4 - y^4 - z^4$ tiene max local $(1,1,1)$

- Hallar $\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$\frac{\partial f}{\partial x}(x,y,z) = 4yz - 4x^3$ $\frac{\partial f}{\partial y}(x,y,z) = 4xz - 4y^3$ $\frac{\partial f}{\partial z}(x,y,z) = 4xy - 4z^3$

- Hallar puntos críticos $\rightarrow$ Hallar $(x_0\, y_0\, z_0) / \nabla f(x_0\, y_0\, z_0) = 0$

$$\begin{cases} 4yz - 4x^3 = 0 \\ 4xz - 4y^3 = 0 \\ 4xy - 4z^3 = 0 \end{cases}$$

Si $(1,1,1)$ es max, entonces $\nabla f(1,1,1) = \vec{0}$

$\xrightarrow{\text{reemplazo}}$ $\nabla f(1,1,1) = (0,0,0)$ ✓

- Hallar $H_f(x,y,z)$

$$H_f(x,y,z) = \begin{pmatrix} -12x^2 & 4z & 4y \\ 4z & -12y^2 & 4x \\ 4y & 4x & -4z^2 \end{pmatrix} \qquad H_f(1,1,1) = \begin{pmatrix} -12 & 4 & 4 \\ 4 & -12 & 4 \\ 4 & 4 & -12 \end{pmatrix}$$

$M_1 = |-12| = -12 < 0$

$M_2 = \begin{vmatrix} -12 & 4 \\ 4 & -12 \end{vmatrix} = 128 > 0$

$M_3 = H_f(1,1,1)$

$$\det(H_f(1,1,1)) = \begin{vmatrix} -12 & 4 & 4 \\ 4 & -12 & 4 \\ 4 & 4 & -12 \end{vmatrix} \begin{matrix} F_2 \rightarrow F_1 + 3F_2 \\ F_3 \rightarrow F_1 + 3F_3 \end{matrix} \begin{vmatrix} -12 & 4 & 4 \\ 0 & -32 & 16 \\ 0 & 16 & -32 \end{vmatrix} = (-1)^{1+1} \cdot (-12) \begin{vmatrix} -32 & 16 \\ 16 & -32 \end{vmatrix} = -9216 < 0$$

Como $\det(M_1) < 0$, $\det(M_2) > 0$, y $\det(H_f(1,1,1)) < 0$, entonces $(1,1,1)$ es un max local

* **Ejercicio 17** Hallar extremos relativos f . def en su dom

ⓐ $f(x,y,z) = -x^3+3x+2y^2+4yz+3y+8z^2$ → $\text{Dom} f = \{(x,y,z) \in \mathbb{R}^3\}$

- Hallar $\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$\frac{\partial f}{\partial x}(x,y,z) = -3x^2+3$ $\quad$ $\frac{\partial f}{\partial y}(x,y,z) = 4y+4z+3$ $\quad$ $\frac{\partial f}{\partial z}(x,y,z) = 4y+16z$

- Hallar puntos críticos $\quad$ $P_1 = \left(1, \frac{-3}{2}, \frac{3}{4}\right)$ $\quad$ $P_2 = \left(-1, \frac{-3}{2}, \frac{3}{4}\right)$

$$\begin{cases} -3x^2+3 = 0 \\ 4y+4z+3=0 \\ 4y+16z = 0 \end{cases} \rightarrow \begin{cases} x^2 = 1 \rightarrow x=1 \lor x=-1 \\ 4y+4z = -3 \\ 4y+16z=0 \end{cases} \rightarrow \left(\begin{array}{cc|c} 4 & 4 & -3 \\ 4 & 16 & 0 \end{array}\right) \xrightarrow{F_2 \to F_2 - F_1} \left(\begin{array}{cc|c} 4 & 4 & -3 \\ 0 & 4 & 3 \end{array}\right)$$

$$\begin{cases} 4y+4z = -3 \rightarrow y = \frac{-6}{4} \rightarrow \left(\frac{-3}{2}\right) \\ 4z = 3 \rightarrow \boxed{z = \frac{3}{4}} \end{cases}$$

- Hallar $H_f(x,y,z)$

$$H_f(x,y,z) = \begin{pmatrix} -6x & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 16 \end{pmatrix}$$

$$H_f(P_1) = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 16 \end{pmatrix} \qquad H_f(P_2) = \begin{pmatrix} -6 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 16 \end{pmatrix}$$

- Chequear P_1 extremo

$\det(M_1(P_1)) = |6| = 6 > 0$

$\det(M_2(P_1)) = \begin{vmatrix} 6 & 0 \\ 0 & 4 \end{vmatrix} = 24 > 0$

$\det(M_3(P_1)) = \begin{vmatrix} 6 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 16 \end{vmatrix} = (-1)^{1+1} \cdot 6 \cdot \begin{vmatrix} 4 & 4 \\ 4 & 16 \end{vmatrix} = 288 > 0$

$P_1 = \left(1, \frac{-3}{2}, \frac{3}{4}\right)$ es un mínimo local

- Chequear P_2 extremo

$\det(M_1(P_2)) = |-6| = -6 < 0$

$\det(M_2(P_2)) = \begin{vmatrix} -6 & 0 \\ 0 & 4 \end{vmatrix} = -2 < 0$

$\det(M_3(P_2)) = \begin{vmatrix} -6 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 16 \end{vmatrix} = (-1)^2 (-6) \begin{vmatrix} 4 & 4 \\ 4 & 16 \end{vmatrix} = -288$

(b) $f(x,y,z) = y + \frac{x}{y} + \frac{z}{x} + \frac{1}{z} \rightarrow \text{Dom} f = \{(x,y,z) \in \mathbb{R}^3\} - \{(0,0,0)\}$

- Hallar $\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$\frac{\partial f}{\partial x}(x,y,z) = \frac{1}{y} - \frac{z}{x^2}$ $\quad \frac{\partial f}{\partial y}(x,y,z) = 1 - \frac{x}{y^2}$ $\quad \frac{\partial f}{\partial z}(x,y,z) = \frac{1}{x} - \frac{1}{z^2}$

- Hallar puntos críticos

$$\begin{cases} \frac{1}{y} - \frac{z}{x^2} = 0 \\ 1 - \frac{x}{y^2} = 0 \\ \frac{1}{x} - \frac{1}{z^2} = 0 \end{cases} \rightarrow \begin{cases} \frac{1}{y} - \frac{z}{x^2} = 0 \\ y^2 = x \\ \frac{1}{x} = \frac{1}{z^2} \rightarrow x = z^2 \end{cases} \rightarrow \left\{ \frac{1}{y} - \frac{z}{(z^2)^2} = 0 \rightarrow \boxed{z^3 = y} \right.$$

(z^2, z^3, z)

$x = z^2$, $x = y^2$, $y = z^3$

$z^2 = y^2$

$z^2 = (z^3)^2 \rightarrow \boxed{z = 1, \vee z = 0}$

Punto $(0,0,0) \notin \text{Dom} f$
$(1,1,1)$

- Hallar $H_f(x,y,z)$

$$H_f(x,y,z) = \begin{pmatrix} \frac{2z}{x^3} & -\frac{1}{y^2} & -\frac{1}{x^2} \\ -\frac{1}{y^2} & \frac{2x}{y^3} & 0 \\ -\frac{1}{x^2} & 0 & \frac{2}{z^3} \end{pmatrix} \qquad H_f(1,1,1) = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

$\det(H_1(1,1,1)) = 2 > 0$

$\det(H_2(1,1,1)) = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3 > 0$

$(1,1,1)$ es menor local

Desarrollo F_3

$$\det(H_3(1,1,1)) = \begin{vmatrix} 2 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{vmatrix} = \underbrace{(-1)^{1+3}(-1)\begin{vmatrix} -1 & -1 \\ 2 & 0 \end{vmatrix}}_{-2} + \underbrace{(-1)^{3+3}\, 2 \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}}_{6} = 4 > 0$$

* Ejercicio 18 Hallar b / $f(x,y) = (b^{-1}-1)(y-2)^2 + (x-1)^2 - 2(y-2)^2$ extr local en $(1,2)$

- Hallar $\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

$\frac{\partial f}{\partial x}(x,y) = 2(x-1)$ $\quad \frac{\partial f}{\partial y}(x,y) = 2(b^{-1}-1)(y-2) - 4(y-2)$

- Hallar puntos críticos $\nabla f(x,y) = \vec{0}$

$$\begin{cases} 2(x-1) = 0 \\ 2(b^{-1}-1)(y-2) - 4(y-2) = 0 \end{cases} \rightarrow \begin{cases} x = 1 \\ (y-2)\left[2(b^{-1}-1) - 4\right] = 0 \rightarrow \boxed{y = 2} \end{cases}$$

- Hallar $H_f(x,y)$

$$H_f(x,y) = \begin{pmatrix} 2 & 0 \\ 0 & 2(b^{-1}-1)-4 \end{pmatrix}$$

para que sea extremo

$\det(H_f(1,2)) > 0$

$2 \cdot \left(\frac{1}{b} - 1\right) - 4 > 0 \rightarrow \left(\frac{1}{b} - 1\right) > 2$

$\boxed{\frac{1}{3} > b}$

si $b > \frac{1}{3}$ entonces $(1,2)$ es un mínimo local

* Ejercicio 20 $\quad C^2,\ f: \mathbb{R}^2 \to \mathbb{R} \quad \underline{y = 3x+2} \to f \circ g = x^2 - \ln|x-1| + 3 \to h(x)$

$\vec{g}(x) = (x, 3x+2)$

Si f tiene extremo en $(2,8)$ $\quad \nabla f(2,8) = \vec{0}$ y $H_f(2,8) > 0$ $\qquad h'(x) = 2x - \frac{1}{x-1}$

$\underbrace{D(f \circ g)(2)}_{h'(2)} = \underbrace{\nabla f(2,8)}_{0} \cdot g'(2) = 0 \qquad h'(2) = 4 - 1 = 3$

$h'(2) = \nabla f(2,8) \cdot g'(2)$ $\to$ si $h'(2) = 3$

$\hookrightarrow 3$ $\quad$ entonces $\nabla f(2,8) \neq 0$ y $g'(2) \neq$. por lo que f no es max en $(2,8)$

* Ejercicio 21 $\quad f(x,y,z) = x^2 + y^2 + z + 2 \quad$ min rel $(-1,1,12)$ cuando esta en punto $\vec{x} = (\mu - 3,\ v+4,\ 2\mu - 2v + 2)$

$\hookrightarrow$ ver si $(-1,1,12)$ pertenece al plano $\vec{x}$

$$\begin{cases} \mu - 3 = -1 \to \mu = 2 \\ v + 4 = 1 \to v = -3 \\ 2\mu - 2v + 2 = 12 \to 4 + 6 + 2 = 12 \end{cases} \to (-1,1,12) \in \vec{x} \ \forall (\mu_0, v_0) = (2,-3)$$

$\hookrightarrow$ Chequear si punto es mínimo rel en f

- Hallar $\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$

$\frac{\partial f}{\partial x}(x,y,z) = 2x \qquad \frac{\partial f}{\partial y}(x,y,z) = 2y \qquad \frac{\partial f}{\partial z}(x,y,z) = 1$

- probar $\nabla f(-1,1,12) = \vec{0}$

$$\begin{cases} 2x \\ 2y \\ 1 \end{cases} \to \begin{cases} 2(-1) = -2 \neq 0 \\ 2 \cdot 1 = 2 \neq 0 \\ 1 \neq 0 \end{cases}$$

$\nearrow$ el punto $(-1,1,12)$ no es pto crítico en f

- Evaluar $(-1,1,12)$ con $h(\mu,v) = f(\vec{x}(\mu,v))$ $\to (\mu,v) = (2,-3)$

$Dh(2,-3) = Df(-1,1,1) \cdot D\vec{x}(2,-3) \qquad \to D_f(x,y,z) = (2x \quad 2y \quad 1)$

$D_f(-1,1,12) = (-2 \quad 2 \quad 1)$

$$Dh(2,-3) = (-2, 2, 1) \underset{1\times3 \quad 3\times2}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2 & -2 \end{pmatrix}} = (0 \quad 0) \qquad D_g(\mu,v) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2 & -2 \end{pmatrix}$$

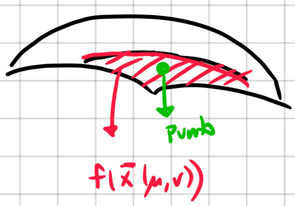
$\downarrow$

Hallar hessiano $h(\mu,v)$

$Dh(\mu,v) = Df(x,y,z) \cdot D\vec{x}(2,-3) \to$ donde $\begin{matrix} x = \mu - 3 \\ y = v + 4 \end{matrix}$

$$Dh(\mu,v) = (2x \quad 2y \quad 1) \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2 & -2 \end{pmatrix} = (2x+2 \quad 2y-2)$$

$$Dh(\mu,v) = (2\mu - 4,\ 2v - 6)$$



$$H_h(\mu,v) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 4 > 0$$ $\to$ el punto $(-1,1,12)$ es mínimo en $f(\vec{x}(\mu,v))$

pero no crítico en f