24/3/2025 傑恩

FUNCIONES VECTORIALES DE UNA VARIABE + CURVAS

* Ejercicio 1 = Analizar existencia de los límites

ⓐ $\lim_{u\to 0}\left(\frac{\operatorname{sen}(u)}{|u|}, \sqrt{u}\right) = \left(\lim_{u\to 0}\frac{\operatorname{sen}(u)}{u}, \lim_{u\to 0}\sqrt{u}\right)$

$\lim_{u\to 0}\frac{\operatorname{sen}(u)}{|u|} = \begin{cases}\lim_{u\to 0^+}\frac{\operatorname{sen}(u)}{u} = 1\\ \lim_{u\to 0^-}\frac{\operatorname{sen}(u)}{-u} = -1\end{cases}$ $\quad \nexists \lim_{u\to 0}\frac{\operatorname{sen}(u)}{u} \Rightarrow \nexists \lim_{u\to 0}\left(\frac{\operatorname{sen}(u)}{|u|}, \sqrt{u}\right)$

ⓑ $\lim_{u\to 0}\left(\frac{e^u-1}{u}, u^2\right) = \left(\lim_{u\to 0}\frac{e^u-1}{u}, \lim_{u\to 0}u^2\right) = (1,0)$

• $\lim_{u\to 0}\frac{\overset{\to 0}{e^u-1}}{\underset{\to 0}{u}} \underset{\overrightarrow{L'H}}{=} \lim_{u\to 0}\frac{e^u}{1} = 1$ • $\lim_{u\to 0}u^2 = 0$

ⓒ $\lim_{t\to 0^+}\left(t\operatorname{sen}\left(\frac{3}{t}\right), 4+t\ln(t)\right) = \left(\lim_{t\to 0^+}t\operatorname{sen}\left(\frac{3}{t}\right), \lim_{t\to 0^+}4+t\ln(t)\right) = (0,4)$

• $\lim_{t\to 0^+}\overset{\to 0}{t}\underbrace{\operatorname{sen}\left(\overset{\to\infty}{\frac{3}{t}}\right)}_{\text{func periódica } [0,1]} = 0$ • $\lim_{t\to 0^+}\left(\overset{\to 4}{4+\underbrace{t\ln(t)}_{\to 0}}\right) = 4$

25/3

* Ejercicio 2 = Analizar continuidad, Determinar si Img es curva, Hallar expr cartesiana de conj Img + GRAFICAR

Hallar el conjunto $\{(x,y) / y = x^2 \dots\}$

ⓐ $\vec{g} = [0,1] \subset \mathbb{R} \to \mathbb{R}^2 / \vec{g}(t) = (t,t^2)$, $g_1 = t$ $g_2 = t^2$

• Continuidad

g_1 continua en intervalo $[0,1]$ y g_2 continua en $[0,1]$ por lo que $\vec{g}$ continua en $[0,1]$

• Det curva ¿Cuándo es curva?

- Función vectorial de una sola var (t)
- Continua → imagen unidimensional
- Derivada $\neq 0$ para todo dominio

¿Es curva?

$\vec{g} = (t,t^2)$, $\vec{g}'(1,2t)$

→ es continua en $[0,1]$

→ $\vec{g}'(1,2t) \neq 0 \quad \forall t \in [0,1]$ → curva regular

→ $\vec{g}$ es inyectiva en $t \in [0,1]$ → curva simple

Conjunto imagen $= \{(t,t^2) / t \in [0,1]\}$

(b) $\vec{g} = [0,1] \subset \mathbb{R} \to \mathbb{R}^2 / \vec{g}(t) = (1-t, (1-t)^2)$ $g_1 = 1-t$ $g_2 = (1-t)^2$

- Hallar continuidad

$g_1 = 1-t \to$ continua $\forall t \in [0,1]$
$g_2 = (1-t)^2 \to$ continua $\forall t \in [0,1]$
$\Big\}$ $\vec{g}$ continua en $[0,1]$

- Determinar si es curva

conj imagen $= \{(1-t, (1-t)^2) / t \in [0,1]\}$

$\to \vec{g}$ continua en $[0,1]$

- No hay puntos repetidos $\to$ curva simple

$\to \vec{g}\,' = (-1, 2t-2) \neq 0$ $\forall t \in$ $\to$ curva regular

(c) $\vec{g} = \mathbb{R} \to \mathbb{R}^2 / \vec{g}(x) = (x, 2x+1)$ $g_1 = x$ $g_2 = 2x+1$

$C_I = \{(x, 2x+1) / x \in \mathbb{R}\}$

- Continuidad

$g_1 = x$ continua $\forall x \in \mathbb{R}$
$g_2 = 2x+1$ continua $\forall x \in \mathbb{R}$
$\Big\}$ $\vec{g}$ continua

- Det curva

$\vec{g}$ continua en $\mathbb{R}$

$\vec{g}\,'(1, 2) \neq 0$ $\forall x \in \mathbb{R} \to$ curva reg.

no hay puntos rep $\to$ curva simple

(d) $\vec{g} = [0,\pi] \subset \mathbb{R} \to \mathbb{R}^2 / \vec{g}(t) = (2\cos(t), 3\,\text{sen}(t))$ $g_1 = 2\cos(t)$ $g_2 = 3\,\text{sen}(t)$

- Continuidad

$g_1 \to$ continua $\forall t \in [0,\pi]$
g_2 continua $\forall t \in [0,\pi]$
$\Big\}$ $\vec{g}$ cont. $\forall t \in [0,\pi]$

- Det curva

$C_I = \{(2\cos(t), 3\,\text{sen}(t)) / t \in [0,\pi]\}$

$\vec{g}$ continua $\forall t \in [0,\pi]$

$\vec{g}\,' = (-2\,\text{sen}(t), 3\cos(t)) \neq \vec{0}$ $\forall t \in [0,\pi]$ $\to$ curva regular

(e) $\vec{g} = \mathbb{R} \to \mathbb{R}^3 / g(t) = (\cos(t), \text{sen}(t), 3t)$ $g_1(t) = \cos(t)$ $g_2(t) = \text{sen}(t)$ $g_3(t) = 3t$

- Continuidad

g_1 continua $\forall t \in \mathbb{R}$
g_2 continua $\forall t \in \mathbb{R}$
g_3 continua $\forall t \in \mathbb{R}$
$\Big\}$ g cont $\forall t \in \mathbb{R}$

- Determinar curva

$\vec{g}$ continua en $\mathbb{R}$

$\vec{g}\,'(t) = (-\text{sen}\,t, \cos t, 3) \neq \vec{0}$ $\forall t \in \mathbb{R}$ $\to$ curva regular

no tiene punto repetido $\to$ curva simple

(f) $\vec{g} = \mathbb{R} \to \mathbb{R}^3 / \vec{g}(t) = (t-1, 2t, t+1)$

- Continuidad

$$\left.\begin{array}{l} g_1(t) = t-1 \to \text{Continua } \forall t \in \mathbb{R} \\ g_2(t) = 2t \to \text{continua } \forall t \in \mathbb{R} \\ g_3(t) = t+1 \to \text{Continua } \forall t \in \mathbb{R} \end{array}\right\} \text{Continua } \forall t \in \mathbb{R}$$

- Determinar curva

$\to \vec{g}$ continua en $\mathbb{R}$

$\to \vec{g}'(1, 2, 1) \neq \vec{0} \ \forall t \in \mathbb{R}$, curva regular

(g) $\vec{g} : [0, 2\pi] \to \mathbb{R}^3 / \vec{g}(t) = (2\,\text{sen}(t), 2\cos(t), 4 - 4\cos(t))$

- Continuidad

$$\left.\begin{array}{l} g_1(t) = 2\,\text{sen}(t) \to \text{cont } \forall t \in [0, 2\pi] \\ g_2(t) = 2\cos(t) \to \text{cont } \forall t \in [0, 2\pi] \\ g_3(t) = 4 - 4\cos(t) \to \text{cont } \forall t \in [0, 2\pi] \end{array}\right\} \vec{g} \text{ cont } \forall t \in [0, 2\pi]$$

- Determinar curva

$\vec{g}$ continua $\forall t \in [0, 2\pi]$

$\vec{g}' = (2\cos(t), -2\,\text{sen}(t), 4 + 4\,\text{sen}(t)) \neq \vec{0} \ \ \forall t \in [0, 2\pi]$

$C_J = \{(x, y, z) /$

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* Ejercicio 3 = Dado los conjuntos, expresar paramétricamente mediante ec vectorial e indicar si parametrización es una curva

27/3

(a) PUNTOS DE $\mathbb{R}^2$

(a_1) $\boxed{y = x^2} \to \vec{g}(x) = (x, x^2) \to$ es una curva $\to$ Def en intervalo $\mathbb{R}$ / continua en int $\mathbb{R}$

(a_2) $y + 2x = 4 \to y = 4 - 2x \to \vec{g} = (x, 4-2x). \to 0 \leq 4 - 2x \leq 2 \to 2 \geq x \geq 1$

$\to$ curva $\vec{g} = (x, 4-2x) \ / \ x \in [1, 2]$

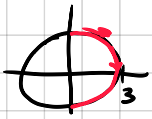
(a_3) $x^2 + y^2 = 9$

$\vec{g}(t) = (3\cos(t), 3\,\text{sen}(t)), \ t \in [0, 2\pi)$ $\to$ curva circunferencia / continua $[0, 2\pi)$, inyectiva

(a_4) $xy = 1 \to y = \frac{1}{x}$ $\quad g(t) = (t, \frac{1}{t})$ Dom $t \in (-\infty, 0) \cup (0, \infty+) \to$ No es curva, discontinua $\vec{g}(t)$ en $t=0$

(a_5) $x^2 + y^2 = 9 \quad x \geq 0$

$\downarrow$ gráficamente



3

$\vec{g}(t) = (3\,\text{sen}(t), 3\cos(t)), \ t \in [0, \pi]$

$\vec{g}(0) = (0, 3)$

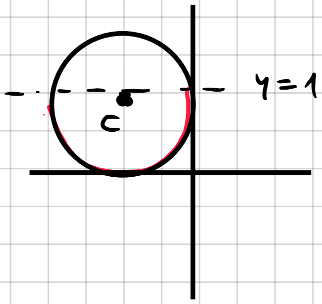
$\vec{g}(\frac{\pi}{2}) = (3, 0)$

$\vec{g}(\pi) = (0, -3)$

Curva $\to$ definida en $t \in [0, \pi]$

$\to$ Continua $\vec{g}(t)$ en $[0, \pi]$

(a6) $x^2+2x+y^2-2y=2 \quad , \quad y\le 1$ $\qquad (x+1)^2+(y-1)^2=4$



→ Completar cuadrados $\quad x^2+2\cdot 1x+1-1+y^2-2\cdot y+1-1=2$ $\qquad$ circunferencia → $c=(-1,1)$, $r=2$

$$(x+1)^2-1+(y-1)^2-1=2$$

$$\vec{g}(t)=(2\cos(t)-1,\ 2\operatorname{sen}(t)+1)\ ,\ t\in[\pi,2\pi)$$

- definida en conjunto
- continua en $[0,2\pi)$

(a7) $9x^2+4y^2=36 \Rightarrow \frac{9x^2}{36}+\frac{4y^2}{36}=1 \Rightarrow \frac{x^2}{4}+\frac{y^2}{9}=1$ → elipse $a=2$, $b=3$

$$\vec{g}(t)=(2\cos(t),\ 3\operatorname{sen}(t)),\ t\in[0,2\pi)$$

(b) PUNTOS DE $\mathbb{R}^3$

(b1) $y=x^2\ ,\ x+z=y\ ,\ z\le 0$

$$\begin{cases} y=x^2\\ x+z=y\\ z\le 0\end{cases} \Rightarrow \begin{cases} z=x^2-x\\ y=x^2\\ z\le 0\end{cases} \quad \vec{g}(t)=(t,t^2,t^2-t) \Rightarrow t^2-t\le 0$$

$t\in[0,1]$ $\qquad t(t-1)\le 0$

$\begin{cases} t\ge 0\\ t-1\le 0\end{cases} \vee \begin{cases} t\le 0\\ t-1\ge 0\end{cases}$ $\quad \begin{cases} t\ge 0\\ t\le 1\end{cases} \vee \begin{cases} t\le 0\\ t\ge 1\end{cases}$ → $[0,1]$, $\emptyset$

curva $\vec{g}(t)$ $\begin{cases} \text{- def intervalo}\\ \text{- continua } \forall t\in[0,1]\end{cases}$

(b2) $x+y=5\ ,\ z=x^2+y^2$

$$\begin{cases} z=x^2+y^2\\ x+y=5\to y=5-x\end{cases} \quad \begin{cases} z=x^2+(5-x)^2\\ y=5-x\end{cases}$$

$z=x^2+5^2-10x+x^2$
$=2x^2-10x+25$

$$\vec{g}(t)=(t,\ 5-t,\ 2t^2-10t+25)$$

Dom $\vec{g}\ \in\mathbb{R}$, $\vec{g}$ continua en $t\in\mathbb{R}$

(b3) $x^2+y^2=4\ ,\ y+2z=2$

$$\begin{cases} x^2+y^2=4\\ y+2z=2\end{cases} \to \begin{cases} x^2+(2-2z)^2=4\\ y=2-2z\end{cases} \to \begin{cases} |x|=4-(2-2z)^2\\ y=2-2z\end{cases} \nearrow \begin{cases} x=4-(2-2z)^2 \text{ si } x\ge 0\\ x=-4+(2-2z)^2 \text{ si } x<0\end{cases}$$

→ estudiar $\boxed{\begin{matrix} z=0\\ z=4\end{matrix}}$

$$\vec{g}(t)=(x(t),\ 2-2t,\ t)$$

estudiar continuidad en $t=0$ y $t=2$

$$\begin{cases} x^2+y^2=4 \longrightarrow (2\cos t,\ 2\operatorname{sen}t,\ 1-\operatorname{sen}(t))\\ y+2z=2 \longrightarrow z=\frac{2-y}{2}=1-\frac{2\operatorname{sen}t}{2}\end{cases}$$

$t\in[0,2\pi)$

$\vec{g}$ continua

DERIVADA FUNCIÓN VECTORIAL ⇒ RECTA TG Y PLANO NORMAL

* Ejercicio 4 : Calcular derivadas

(a) $\vec{\rho}'(0)$ si $\vec{\rho}(t) = (2\,\text{sen}\,t,\ t\cos t),\ t\in\mathbb{R} \Rightarrow \vec{\rho}'(t) = (2\cos(t),\ \cos t - t\,\text{sen}\,t)$

$$\vec{\rho}'(0) = (2\cos(0),\ \cos(0) - 0\cdot\text{sen}(0)) = (2,1)$$

(b) $\vec{\sigma}'(0)$ si $\vec{\sigma}(t) = \begin{cases} \left(\dfrac{\text{sen}(t)}{t},\ 2e^t\right) & \text{si } t\neq 0 \\ (1,2) & \text{si } t=0 \end{cases}$ $\qquad \vec{\sigma}_1 = \dfrac{\text{sen}\,t}{t} \qquad \vec{\sigma}_2 = 2e^t$

$$\vec{\sigma}'(0) = \lim_{h\to 0} \frac{\vec{\sigma}(0+h) - \vec{\sigma}(0)}{h}$$

- $\displaystyle \lim_{h\to 0} \frac{\sigma_1(0+h) - \sigma_1(0)}{h} = \lim_{h\to 0} \frac{\frac{\text{sen}(0+h)}{0+h} - \sigma_1(0)}{h} = \lim_{h\to 0} \frac{\left(\frac{\text{sen}(h)}{h} - 1\right)}{h} = \lim_{h\to 0} \frac{\left(\frac{\text{sen}\,h - h}{h}\right)}{\left(\frac{h}{1}\right)}$

$$= \lim_{h\to 0} \frac{\text{sen}(h) - h}{h^2} = \lim_{h\to 0} \frac{\cos(h) - 1}{2h} = \lim_{h\to 0} \frac{-\text{sen}(h)}{2} = 0$$

$$\vec{\sigma}'(0) = (0,2)$$

- $\displaystyle \lim_{h\to 0} \frac{\sigma_2(h+0) - \sigma_2(0)}{h} = \lim_{h\to 0} \frac{2e^h - 2}{h} \underset{\text{L'Hop}}{=} \lim_{h\to 0} \frac{2e^h}{1} = 2$

(c) $\vec{\rho}'(\pi)$, si $\vec{\rho}(t) = \left(\dfrac{\text{sen}(2t)}{t-\pi},\ \cos(t/2)\right)$ con $t\in\mathbb{R} - \{\pi\}$, $\vec{\rho}(\pi) = (2,0)$ $\qquad \rho_1 = \dfrac{\text{sen}(2t)}{t-\pi} \qquad \rho_2 = \cos(t/2)$ $\qquad \rho'(\pi) = (\rho_1'(\pi), \rho_2'(\pi))$

• coc. incremental $\displaystyle \rho_1'(\pi) = \lim_{h\to 0} \frac{\rho(\pi+h) - \rho(\pi)}{h} = \lim_{h\to 0} \frac{\frac{\text{sen}(2(h+\pi))}{h+\pi-\pi} - 2}{h} = \lim_{h\to 0} \frac{\frac{\text{sen}(2h+2\pi)}{h} - \frac{2h}{h}}{h}$

$$\lim_{h\to 0} \frac{\text{sen}(2h+2\pi) - 2h}{h^2} \underset{\text{L'H}}{=} \lim_{h\to 0} \frac{2\cos(2h+2\pi) - 2}{2h} \underset{\text{L'H}}{=} \lim_{h\to 0} \frac{-4\,\text{sen}(2h+2\pi)}{2} = \lim_{h\to 0} -2\,\text{sen}(2h+2\pi) = 0$$

⇒ coc. increm. $\displaystyle \rho_2'(\pi) = \lim_{h\to 0} \frac{\rho_2(\pi+h) - \rho_2(\pi)}{h} = \lim_{h\to 0} \frac{\cos\left(\frac{\pi+h}{2}\right)}{h} = \lim_{h\to 0} -\text{sen}\left(\frac{\pi+h}{2}\right)\cdot\frac{1}{2} = -\frac{1}{2}$

$$\vec{\rho}'(\pi) = \left(0, -\frac{1}{2}\right)$$

ⓓ $\vec{\sigma}'(0)$ si $\vec{\sigma}(x) = \begin{cases} (x, x^2) & \text{si } x \geq 0 \\ (x, x^2+1) & \text{si } x < 0 \end{cases}$ $\qquad \vec{\sigma}'(0) = (\sigma_1'(0), \sigma_2'(0))$ $\qquad \sigma_1 = x$, $\quad \sigma_2 = \begin{cases} x^2 & \text{si } x \geq 0 \\ x^2+1 & \text{si } x < 0 \end{cases}$

∘ chequear continuidad en $x = 0$

$\Rightarrow \vec{\sigma}(0) = (0, 0) \longrightarrow \exists\, \vec{\sigma}(0)$

$\Rightarrow \lim\limits_{x\to 0} \vec{\sigma}(x) = \lim\limits_{x\to 0} (\sigma_1(x), \sigma_2(x)) = \begin{cases} \lim\limits_{x\to 0} \sigma_1(x) = \lim\limits_{x\to 0} x = 0 \\ \lim\limits_{x\to 0} \sigma_2(x) = \begin{cases} \lim\limits_{x\to 0^+} x^2 = 0 \\ \lim\limits_{x\to 0^-} x^2+1 = 1 \end{cases} \end{cases}$

$\lim\limits_{x\to 0^+} \vec{\sigma}(x) = (0,0)$

$\lim\limits_{x\to 0^-} \vec{\sigma}(x) = (0,1)$

$\nexists \lim\limits_{x\to 0} \vec{\sigma}(x)$

$\nexists \lim\limits_{x\to 0} \vec{\sigma}(x) \longrightarrow$ como no existe límite, $\vec{\sigma}(x)$ no es continua en $\vec{\sigma}(0)$ y por lo tanto no es derivable $\nexists\, \vec{\sigma}'(0)$

✱ Ejercicio 5 $\quad$ C curva, param $\vec{\sigma}(t) = (R\cos(t), R\operatorname{sen}(t))$, $t \in [0, 2\pi)$, $R > 0$

hallar rect tg en $\vec{\sigma}(\pi/4) \longrightarrow$ Graf curva, tg y sentido de recorrido

∘ hallar $\sigma'(t)$

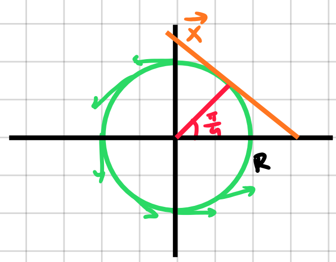
$\sigma'(t) = (-R\operatorname{sen}(t), R\cos(t))$

$\sigma'(\frac{\pi}{4}) = (-R\operatorname{sen}(\frac{\pi}{4}), R\cos(\frac{\pi}{4})) = \left(-\frac{R\sqrt{2}}{2}, \frac{R\sqrt{2}}{2}\right)$

• hallar $\sigma(\frac{\pi}{4}) \to \left(\frac{R\sqrt{2}}{2}, \frac{R\sqrt{2}}{2}\right)$

∘ Armar ec rect. tg $\sigma(\frac{\pi}{4})$

$\vec{X} = \lambda\left(-\frac{R\sqrt{2}}{2}, \frac{R\sqrt{2}}{2}\right) + \left(\frac{R\sqrt{2}}{2}, \frac{R\sqrt{2}}{2}\right) \to$ ec rect tg $\sigma(\frac{\pi}{4})$



✱ Ejercicio 6 $\quad \vec{\sigma}(t) = (\cos(t), \operatorname{sen}(t))$, $t \in [0, 2\pi)$ $\quad$ Hallar puntos (x,y) donde rect tg $\parallel$ $x + 2y = 1$

• Hallar parametrización recta $x + 2y = 1$

$x + 2y = 1 \Rightarrow y = \frac{1-x}{2} \quad \Rightarrow \vec{r}(x) = \left(x, \frac{1-x}{2}\right) = \underbrace{x\left(1, -\tfrac{1}{2}\right) + (0,1)}_{\text{Ec parametrizada de la recta}}, \; x \in \mathbb{R}$

$\left(1, -\tfrac{1}{2}\right) \to$ vector director

los puntos son: $\sigma(t) \,/\, t = \arctan(2) \lor t = \arctan(2) + \pi$

• Hallar $\sigma'(t)$

$\sigma'(t) = (-\operatorname{sen}(t), \cos(t))$

• Hallar $t \,/\, \sigma'(t) = (1, -\frac{1}{2})$

$\vec{\sigma}(t) = (-\operatorname{sen}(t), \cos(t)) = (1, -\frac{1}{2}) \Rightarrow$ puede ser un múltiplo de $(1, -\frac{1}{2})$

$\vec{\sigma}'(t) = (-\operatorname{sen}(t), \cos(t)) = \lambda(2, -1)$

$\begin{cases} -\operatorname{sen}(t) = 2\lambda \\ \cos(t) = -\lambda \Rightarrow -\cos t = \lambda \end{cases}$

$\begin{cases} -\operatorname{sen}(t) = 1 \\ \cos(t) = -\frac{1}{2} \end{cases} \rightarrow \left\{ \cancel{t \neq 0 \;\wedge\; t = \pi \quad t \neq 0{,}47 \text{ rad}} \right.$

$\begin{cases} t_1 = \arctan(2) \\ t_2 = \arctan(2) + \pi \end{cases}$

$\Downarrow$

recordar circulo trigonométrico

$\begin{cases} \operatorname{sen}(t) = 2\cos(t) \\ \lambda = -\cos(t) \end{cases}$

$\frac{\operatorname{sen}(t)}{\cos(t)} = tg(t) = 2$

* Ejercicio 7

$\vec{\sigma}(t) = (t^2, t^3+1, t^3-1)$, $t \in [1,4]$

(a) Hallar rect tg y plano normal en $(4,9,7)$

- Hallar t en $(4,9,7)$

$$\begin{cases} t^2 = 4 \\ t^3+1 = 9 \\ t^3-1 = 7 \end{cases} \Rightarrow \begin{cases} t=2 \ \cancel{\vee\ t=-2} \\ t=2 \\ t=2 \end{cases}$$

- Hallar $\vec{\sigma}'(t)$

$\vec{\sigma}'(t) = (2t, 3t^2, 3t^2)$

$\vec{\sigma}'(2) = (4, 12, 12)$

- recta tg $= \lambda(1,3,3) + (4,9,7)$

- plano normal

$N = (1,3,3)$

$(1,3,3)\,[(x,y,z) - (4,9,7)] = 0$

(b) Hallar módulo vect tg $\sigma'(t_0)$, $\vec{\sigma}(t_0) = (4,9,7)$

$$\|\vec{\sigma}'(2)\| = \|(4,12,12)\| = \sqrt{4^2+12^2+12^2} = 4\sqrt{19}$$

(c) Verificar curva es plana

↳ los vect tg def en $[1,2]$ pertenecen al plano

↳ Hallar ec plano y verificar si aplica para ec curva

⇒ Hallar dos vectores tangentes

$\sigma'(1) = (2,3,3)$

$\sigma'(2) = (4,12,12) \Rightarrow (1,3,3)$

- Hallar $\vec{N}$ plano de la curva

$$\vec{N} = (2,3,3)\times(1,3,3) = \begin{vmatrix} i & j & k \\ 2 & 3 & 3 \\ 1 & 3 & 3 \end{vmatrix} = i(0) - j(3) + k(3)$$
$$= (0,-3,3) \Rightarrow (0,-1,1)$$

- Ec plano $= (0,-1,1)[(x,y,z)-(1,2,0)] = 0$

$z - y + 2 = 0 \Rightarrow \boxed{y - z = 2}$

- Verificar si ec curva está en plano

$(t^3+1) - (t^3-1) = t^3+1-t^3+1 = 2$ ✓ curva en plano

(d) Hallar intersección de C con ecuación $y+z=2$

$$C = \begin{cases} x = t^2 \\ y = t^3+1 \\ z = t^3-1 \end{cases} \qquad C \cap \Pi = (t^3+1) + (t^3-1) = 2$$

$2t^3 = 2 \Rightarrow t^3 = 1 \Rightarrow t = 1$

- hallar punto de intersección

$\vec{\sigma}(1) = (1,2,0)$

(e) Hallar puntos C / rect tg ⊥ plano $x+9y+3z=0$

N plano $\to \lambda(1,3,9)$, $\lambda \in \mathbb{R}$ → la $\vec{N}$ es ortog al plano

- Hallar t / $\vec{\sigma}'(t) = \lambda(1,3,9)$

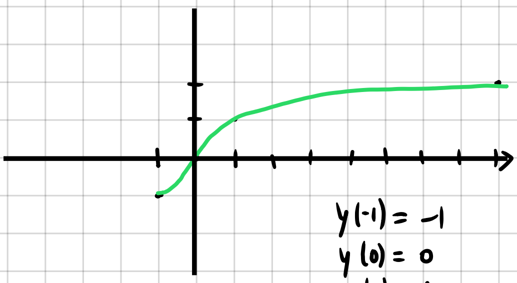
$$\begin{cases} 2t = \lambda \\ 3t^2 = 3\lambda \\ 3t^2 = 9\lambda \end{cases} \Rightarrow \begin{cases} t = \frac{1}{2}\lambda \\ t^2 = \lambda \\ t^2 = 3\lambda \end{cases} \Rightarrow \boxed{t=0 \ \wedge\ \lambda=0}$$

no existen puntos, $t \in [1,4]$ que tenga recta tg ortogonal al plano $x+9y+3z=0$

30/3/25

*** Ejercicio 8** $\vec{x} = (t, \sqrt[3]{t})$, $t \in [-1, 8]$

ⓐ Gráfico $\mathbb{R}^2$ $y = \sqrt[3]{x}$, $x \in [-1, 8]$



$y(-1) = -1$
$y(0) = 0$
$y(1) = 1$

ⓑ Hallar tangente y normal en $(1,1)$

∘ Hallar t / $\vec{x} = (1,1)$

$\begin{cases} t = 1 \\ \sqrt[3]{t} = 1 \end{cases} \Rightarrow \boxed{t = 1}$ ⟶ ∘ Hallar $\vec{x}\,'(t)$

$\vec{x}\,'(t) = \left(1, \frac{1}{3\sqrt[3]{t^2}}\right)$

• Hallar vector tg

$\vec{x}\,'(1) = \left(1, \frac{1}{3}\right)$

→ Hallar recta tg

$\vec{x}_1 = \lambda\left(1, \frac{1}{3}\right) + (1,1)$, $\lambda \in \mathbb{R}$

⇒ Hallar vector normal ⟶ $\vec{v} \perp \left(1, \frac{1}{3}\right)$

$(x,y)\left(1, \frac{1}{3}\right) = 0$

$x + \frac{1}{3}y = 0 \Rightarrow \begin{cases} x = \frac{1}{3} \\ y = -1 \end{cases}$

$\vec{v} = \left(\frac{1}{3}, -1\right)$

→ recta normal = $\vec{x}_2 = \mu\left(\frac{1}{3}, -1\right) + (1,1)$, $\mu \in \mathbb{R}$

ⓒ Determinar recta tg en $(0,0)$

$\vec{x}\,'(0) = \left(1, \frac{1}{3}\frac{1}{\sqrt[3]{t^2}}\right) \Rightarrow t \neq 0$

↳ en esta parametrización no admite tg en $(0,0)$ $\nexists\, \vec{x}\,'(0)$

ⓓ $\vec{x} = (u^3, u)$ $u \in [-1, 2]$

$\vec{x}\,'(0) = (3 \cdot 0^2, 1) = (0,1) \Rightarrow$ como $\exists\, \vec{x}\,'(0) \neq \vec{0}$ admite recta tg en $(0,0)$

recta tg en $(0,0)$ = $r(\lambda) = \lambda(0,1)$, $\lambda \in \mathbb{R}$

*** Ejercicio 9** $\vec{x} = (\underbrace{\sqrt{5}\cos(t), \sqrt{5}\operatorname{sen}(t)}_{g(t)})$, $t \in \mathbb{R}$

Sea $\vec{g}(t)$ pos
$\vec{g}\,'(t)$ vel
$\vec{g}\,''(t)$ acel

pos $\vec{g}(t) = (\sqrt{5}\cos(t), \sqrt{5}\operatorname{sen}(t))$ vel $\vec{g}\,'(t) = (-\sqrt{5}\operatorname{sen}(t), \sqrt{5}\cos(t))$ acel $\vec{g}\,''(t) = (-\sqrt{5}\cos(t), -\sqrt{5}\operatorname{sen}(t))$

ⓐ Calc y graf $\vec{v}$ y $\vec{a}$ cuando $\vec{x} = \vec{g}(t) = (1,2)$

∘ Hallar t cuando $\vec{g}(t) = (1,2)$ ⟶ ojo función periódica

$\begin{cases} \sqrt{5}\cos(t) = 1 \\ \sqrt{5}\operatorname{sen}(t) = 2 \end{cases} \Rightarrow \begin{cases} t = \arccos\left(\frac{1}{\sqrt{5}}\right) + \left(2\pi - \arccos\left(\frac{2}{\sqrt{5}}\right)\right)k, & k \in \mathbb{N} \\ t = \operatorname{arcsen}\left(\frac{2}{\sqrt{5}}\right) + \left(\pi - \operatorname{arcsen}\left(\frac{2}{\sqrt{5}}\right)\right)k, & k \in \mathbb{N} \end{cases}$ pero periódicamente sólo coincide en primer cuadrante

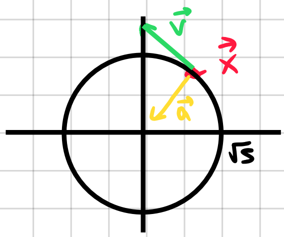
$\boxed{t = \arccos\left(\frac{1}{\sqrt{5}}\right) = \operatorname{arcsen}\left(\frac{2}{\sqrt{5}}\right)}$

$t = \arccos\left(\frac{1}{\sqrt{5}}\right) + 2\pi k$, $k \in \mathbb{N}$
$= \operatorname{arcsen}\left(\frac{2}{\sqrt{5}}\right) + 2\pi k$, $k \in \mathbb{N}$

• Hallar $\vec{v}$ y $\vec{a}$

$\vec{v} = \vec{g}\,'\left(\arccos\left(\frac{1}{\sqrt{5}}\right)\right) = (-2, 1)$ // $\vec{a} = \vec{g}\,''\left(\arccos\left(\frac{1}{\sqrt{5}}\right)\right) = -2, -1$

• GRÁFICO (MCU)



• $\vec{v}$ const y $\vec{a}$ const

$\|\vec{v}\| = \sqrt{(-\sqrt{5}\operatorname{sen}(t))^2 + (\sqrt{5}\cos(t))^2} = \sqrt{5(\operatorname{sen}^2(t) + \cos^2(t))} = \sqrt{5}$ m/s

$\|\vec{a}\| = \sqrt{(-\sqrt{5}\cos(t))^2 + (-\sqrt{5}\operatorname{sen}(t))^2} = \sqrt{5(\cos^2(t) + \operatorname{sen}^2(t))} = \sqrt{5}$ m/s²

módulo constante, lo que cambia es la dirección del vector $\vec{v}$ y $\vec{a}$

* Ejercicio 10 = Hallar parametrización regular de curva, y recta tg en punto indicado

ⓐ $y = 4-x$, $z = 4-x^2$, $A = (1,3,3)$

param $\vec{g}(t) = (t, 4-t, 4-t^2) \Rightarrow \vec{g}'(t) = (1, 1, 2t) \Rightarrow \vec{g}'(1) = (1,1,2)$

→ recta tg a curva en $A = \lambda(1,1,2) + (1,3,3)$

ⓑ $x^2+y^2+z^2 = 2$, $z = \sqrt{x^2+y^2}$, $A = (0,1,1)$

$x^2+y^2+\left(\sqrt{x^2+y^2}\right)^2 = 2$ $\boxed{z^2>0}$

$x^2+y^2+x^2+y^2 = 2$ $\boxed{z=\sqrt{1}=1}$

$2(x^2+y^2) = 2 \Rightarrow \boxed{x^2+y^2=1}$

• parametrización $\vec{g}(t) = (\cos(t), \operatorname{sen}(t), 1)$

• derivada $\vec{g}'(t) = (-\operatorname{sen}(t), \cos(t), 0)$

• Hallar t / $\vec{g}(t) = (0,1,1)$

$$\begin{cases} \cos(t) = 0 \\ \operatorname{sen}(t) = 1 \\ 1 = 1 \end{cases} \Rightarrow t = \frac{\pi}{2} + 2\pi k,\ k \in \mathbb{R}$$

$\vec{g}'\left(\frac{\pi}{2}\right) = \left(-\operatorname{sen}\left(\frac{\pi}{2}\right), \cos\left(\frac{\pi}{2}\right), 0\right) = (-1,0,0)$

∘ recta tg a $A = \lambda(-1,0,0) + (0,1,1)$

ⓒ $z = x+y^2$, $x = y^2$, $A = (4,2,8)$

$$\begin{cases} z = 2y^2 \\ x = y^2 \end{cases} \Rightarrow \text{parametrización} \quad \vec{\sigma}(t) = (t^2, t, 2t^2)$$

$\vec{\sigma}'(t) = (2t, 1, 4t)$

• Hallar $\vec{\sigma}'(2)$

$\vec{\sigma}'(2) = (4, 1, 8)$

recta tg a $A = \mu(4,1,8) + (4,2,8)$, $\mu \in \mathbb{R}$

ⓓ $x^2+y^2+z^2 = 6$, $z = x^2+y^2$, $A = (1,1,2)$

$$\begin{cases} x^2+y^2+z^2 = 6 \\ z = x^2+y^2 \end{cases} \Rightarrow \begin{cases} x^2+y^2+(x^2+y^2)^2 = 6 \\ z = x^2+y^2 \end{cases} \Rightarrow x^2+y^2+(x^2+y^2)^2 = 6$$

$\cancel{x^2+y^2+x^4+2x^2y^2+y^4 = 6}$

$\cancel{x^4+x^2+2x^2y^2+y^4+y^2 = 6}$

$\cancel{a^2+a+2ab+b^2+b = 6}$

$\cancel{a = x^2}$
$\cancel{b = y^2}$

$x^2+y^2+(x^2+y^2)^2 = 6$

$(r_0\cos(t))^2+(r_0\operatorname{sen}(t))^2+\left(r_0^2\cos(t)^2+r_0^2\operatorname{sen}(t)^2\right)^2 = 6$

$r_0^2(\cos^2(t)+\operatorname{sen}^2(t)) + \left(r_0^2(\cos^2(t)+\operatorname{sen}^2(t))\right)^2 = 6$

$r_0^2+(r_0^2)^2 = 6$

$r_0^2+r_0^4-6 = 0$ $\quad a = r_0^2$

$a^2+a-6 = 0$ ↗ $a_1, a_2 = \frac{-1\pm\sqrt{1^2-4(-6)}}{2} =$ $a_1 = 2$, $a_2 = -3$ (x)

$a_1 = r_0^2 = 2$

$|r_0| = \sqrt{2}$ ↗ $r_0 = \sqrt{2}$, $r_0 = -\sqrt{2}$ (x) → $r_0 > 0$

param $\vec{g}(t) = \left(\sqrt{2}\cos(t), \sqrt{2}\operatorname{sen}(t), \left(\sqrt{2}\cos(t)+\sqrt{2}\operatorname{sen}(t)\right)^2\right)$

$\vec{g}(t) = (\sqrt{2}\cos(t), \sqrt{2}\operatorname{sen}(t), 2)$

↓

derivada $\vec{g}'(t) = (-\sqrt{2}\operatorname{sen}(t), \sqrt{2}\cos(t), 0)$

⇒ Recta tg en A → $\vec{g}(t) = A$, $t = \frac{\pi}{4}$ (45°) $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$g'\left(\frac{\pi}{4}\right) = \left(-\frac{\sqrt{2}\sqrt{2}}{2}, \frac{\sqrt{2}\sqrt{2}}{2}, 0\right) = (-1,1,0)$

recta tg $= \vec{r}(\lambda) = \lambda(-1,1,0) + (1,1,2)$

* Ejercicio 11

$$\begin{cases} x = 1 \\ y = t \\ z = t^3 + 13 \end{cases}$$

(a) $z = x^4 + xy^3 + 12$, $x = 1$

$$\begin{cases} z = x^4 + xy^3 + 12 \\ x = 1 \end{cases} \Rightarrow \begin{cases} z = y^3 + 13 \\ x = 1 \end{cases} \Rightarrow \text{parametrización} = \vec{g}(t) = (1, t, t^3+13)$$

$$\vec{g}\,'(t) = (0, 1, 3t^2)$$

- Hallar t cuando $\vec{g}(t) = (1, -2, 5) \Rightarrow \boxed{t = -2}$
- Hallar $\vec{g}\,'(-2) = (0, 1, 12)$
- Ecuación recta tg en $(1,-2,5) \Rightarrow \vec{r}(\lambda) = \lambda(0,1,12) + (1,-2,5)$, $\underline{\underline{\lambda \in \mathbb{R}^+}}$ → λ positivo porque y es creciente
- Hallar λ cuando $\vec{r}(\lambda)$ intersecciona a $y = 1$

$y = \lambda - 2 = 1 \rightarrow \boxed{\lambda = 3}$

- Hallar punto $\vec{r}(3) = (0, 3, 36) + (1, -2, 5) = \underline{\underline{(1, 1, 41)}}$

La abeja corta al plano $y = 1$ en el punto $(1, 1, 41)$

(b) $\vec{r}(t) = (t - \operatorname{sen}(t), 1 - \cos(t))$, $t \in \mathbb{R}$

→ velocidad $= \vec{r}\,'(t) = (1 - \cos(t), \operatorname{sen}(t))$ → función rapidez $= g(t) = \sqrt{(1-\cos(t))^2 + \operatorname{sen}^2(t)}$

→ aceleración $= \vec{r}\,''(t) = (\operatorname{sen}(t), \cos(t))$ → función modulo acel $= h(t) = \sqrt{\cos^2(t) + \operatorname{sen}^2(t)} = 1$

- hallar máxima velocidad (rapidez)

$$g'(t) = \left(\sqrt{(1-\cos t)^2 + \operatorname{sen}^2 t}\right)' = \frac{d}{dx}\left(\left(1^2 - 2\cos t + \underbrace{\cos^2 t + \operatorname{sen}^2 t}_{1}\right)^{\frac{1}{2}}\right)$$

$$= \frac{d}{dx}\left(\left(2 - 2\cos t\right)^{\frac{1}{2}}\right) = \frac{1}{2}\left(2 - 2\cos t\right)^{-\frac{1}{2}} \cdot 2\operatorname{sen} t = \frac{\operatorname{sen}(t)}{\sqrt{2 - 2\cos(t)}} \quad , \ t \neq 0 + 2\pi k$$

- hallar t cuando $g'(t) = 0$ → extremo rel. $|\vec{v}|$ → chequear máximo o mínimo (alta paja, skip)

$$g'(t) = \frac{\operatorname{sen}(t)}{\sqrt{2 - 2\cos(t)}} \rightarrow \operatorname{sen}(t) = 0 \rightarrow \underset{\notin \text{Dom}}{\cancel{t = 0}} \ \vee \ \boxed{t = \pi}$$

$$g(\pi) = \sqrt{(2 - 2\cos(\pi))} = 2 \rightarrow \text{max vel}$$

$$g(0) = \sqrt{(2 - 2\cos(0))} = 0 \rightarrow \text{min vel}$$

rapidez máxima $\|\vec{v}\| = 2$

mod aceleración constante $|\vec{a}| = 1$

c - particula se mueve en $y^2 = 2x \longrightarrow \vec{r}(t) = \left(\frac{1}{2}t^2, t\right)$ $\nearrow t = 4$

- rapidez constante $5\ m/s$

• hallar velocidad $\vec{v}(t) = \vec{r}\,'(t)$

$\vec{v}(t) = \vec{r}\,'(t) = \left(t, 1\right) \longrightarrow |\vec{v}(2)| = \sqrt{2^2+1} = \sqrt{5}$ $\nearrow$ no es vel $5 m/s$

• hallar t cuando pasa por $(2,2) \rightarrow \vec{r}(t) = (2,2)$

$\vec{r}(t) = \left(\frac{1}{2}t^2, t\right) = (2,2)$

$\begin{cases} t^2 = 4 \\ t = 2 \end{cases} \rightarrow \begin{cases} t = 2 \vee t = -2 \\ \underline{\underline{t = 2}} \end{cases}$

• hallar otra parametrización

$\vec{v}(t) = (t, 1) = t'(t, 1)$

$|\vec{v}(t)| = \sqrt{(t't)^2 + (t')^2} = \sqrt{t'^2(t+1)} = |t'|\sqrt{t+1} = t'\sqrt{t+1} = 5\ m/s \Rightarrow t' = \frac{5}{\sqrt{t+1}}$

$\vec{v}(t) = t'(t,1) = \frac{5}{\sqrt{t^2+1}}(t,1)$

hallar $\vec{v}(2) = \frac{5}{\sqrt{5}}(2, 1) = \left(\frac{10}{\sqrt{5}}, \frac{5}{\sqrt{5}}\right)$